General Gauge Principle

2017/3/27,4/5,10

This is a guidance of monumental thesis(1956,1967).The aim is deriving **Lagrangean**

$\mathcal{L}_G \equiv -c\bar{\phi}(\hbar\gamma^{\mu}(\partial_{\mu}-A^a_{\mu}(x)G_a)+mc)\phi-\frac{1}{4}(\partial_{\mu}A^a_{\nu}-\partial_{\nu}A^a_{\mu}-f_b{}^a{}_c A^b_{\mu}A^c_{\nu})^2$

$+(ic)^{-1}B^a\partial_{\mu}A^a_{\mu}+\frac{1}{2}\alpha B^aB^a-\chi\,\partial_{\mu}\bar{C}^a\cdot D_{\mu}C^a$. ＊$\langle D_{\mu}C^a=\partial_{\mu}C^a+f_b{}^a{}_c C^b A^c_{\mu}\rangle$

with interaction force$=j^a_{\mu}A^a_{\mu}$ type<Noether current with gauge field>.

＊R.Utiyama,Phys.Rev.**101**(1956),1597<**General Gauge Field Theory**>.

＊＊L.D.Faddeev and V.N.Popov:Pphys Lett.25B(1967)29.<**Quantization of Gauge Field** >.

[1]:Introduction to General Gauge Principle.

Physics(=Lagrangean)is invariant by localized Gauge Transform.

Physical Objects is measured by various gauge.Then essense of physics never depend on those gauge<shifting coordinates paralell or rotational. $\phi\to\exp(\varepsilon^a(x)G_a)\phi$.>.

☞:Lorentz Invariant in special theory of relativity demands **invariant of dynamic equation** (Euler ones) in **global** coordinate transform(**free particles with nothing interaction force**).

While **localized** gauge invariant demands each different ways at every each time and space point.That is,it is no more global space uniformity,but it is local uniformity,or something non uniform motion in global(not free particle with constant velocity,but **accelerated motion** caused by **interaction force**).

In the below,we show **emerging *gauge field*** in covariant derivative by simple way.

(1)**localized gauge transform**.<Einstein convention:$A^{\mu}B_{\mu}\equiv\Sigma_{\mu}A^{\mu}B_{\mu}$:sum on repeated suffix>

Physics is invariant by parallel shifting(coordinate rotation).

(a) $\delta\phi(x)\equiv\varepsilon^a(x)G_a\phi(x)$. . . . . .original definition of localized gauge transform.

(b) $\varepsilon^a(x)$ ≡infinitesimal parameter depending on time and space($x\equiv x_{\mu}\equiv(ict,x_1,x_2,x_3)$).

(c)G_a≡matrix of constant components of the Lie algebra.

(d)Lie Algebra definition：$[G_p,G_q]\equiv G_pG_q-G_qG_p=f_p{}^r{}_qG_r=\Sigma_r f_p{}^r{}_qG_r$. matrix representation.

$*f_b{}^a{}_c=-f_c{}^a{}_b=-f_a{}^b{}_c=-f_b{}^c{}_a$. 〈☞: $\phi\equiv(\phi_1,\phi_2,\ldots,\phi_N)^t$.t=transposing〉

(2)**definition of covariant derivative.**<☞: $\partial_{\mu}\phi(x)\equiv\partial_{\mu}\phi(x)/\partial x_{\mu}$.>

(a)definition of parallel shifting(coordinate rotation)...Physics is invariant by this !!.

$\phi(x+\Delta x)_{//}\equiv\phi(x)+\varepsilon^a(x)G_a\phi(x)=\phi(x)+\Delta x_{\mu}\cdot\partial_{\mu}\varepsilon^a(x)G_a\phi(x)$.

(b)**Invariantization of derivative＝the definition of covarinat derivative.**

$\lim_{\Delta x\to 0}[\phi(x+\Delta x)-\phi(x+\Delta x)_{//}]/\Delta x=\partial_{\mu}\phi(x)-A^a_{\mu}(x)G_a\phi(x)\equiv D_{\mu}\phi(x)$.

(3)**Localized gauge transform and emerging gauge field**$=A^a_{\mu}(x)$.

$\to\ \partial_{\mu}\varepsilon^a(x)\equiv A^a_{\mu}(x)$。〈$A^a_{\mu}$ the first constrain condition from TR parameter$=\varepsilon^a$〉.

(4)**Spinor Field Lagrangean.** 〈☞ : $\bar{\phi}=\phi^{*t}\gamma^{o}$.　＊t＝conjugate and transpose〉

(a)Free Field Spinor Lagrangean : $\mathcal{L}_{\phi}(\phi, \partial_{\mu}\phi) = -c\phi(\hbar\gamma^{\mu}\partial_{\mu} + mc)\phi$.

(b)Non Free Field Spinor Lagrangean :

$\mathcal{L}_{S}(\phi, D_{\mu}\phi) = -c\bar{\phi}(\hbar\gamma^{\mu}D_{\mu} + mc)\phi = -c\bar{\phi}(\hbar\gamma^{\mu}(\partial_{\mu} - A^{a}_{\mu}(x)G_{a}) + mc)\phi$.

☞ : we assume independent variation of $\bar{\phi}=\phi^{*t}\gamma^{o}$ and ϕ in above $\mathcal{L}_{S}$.

Euler Equation: $0 = \partial\mathcal{L}_{S}/\partial\bar{\phi} = -c(\hbar\gamma^{\mu}(\partial_{\mu} - A^{a}_{\mu}(x)G_{a}) + mc)\phi$.

(c)spinor(**matter**)×guage fields mutual interaction。

$\mathcal{L}_{I} = -c\hbar\bar{\phi}\gamma^{\mu}A^{a}_{\mu}(x)G_{a}\phi \equiv j^{a}_{\mu}(x)A^{a}_{\mu}(x)$.

(d)**Noether Current** : $j^{a}_{\mu}(x) \equiv -c\hbar\bar{\phi}\gamma^{\mu}G_{a}\phi$.

{ϕ (**matter**) and A^{a}_{μ} (**force**)} are main casts in quantum reaction.

Also **dipole field of vacuum** must have similar reaction (→**[5]**)

[2] : Guage Field Transform.

$\mathcal{L}_{\phi}(\phi, \partial_{\mu}\phi) = -c\bar{\phi}(\hbar\gamma^{\mu}\partial_{\mu} + mc)\phi$.

Global Lorentz Covariant yield above free field spinor Lagrangean.In order to accomplish complete **localized Gauge Covariant** of $\mathcal{L}_{S}(\phi, D_{\mu}\phi)$, the Lagrangean must be invariant by both variation of (ϕ, A^{a}_{μ}).Then transform of δA^{a}_{μ} is determined as follows.

$\delta A^{a}_{\mu}(x) = \partial_{\mu}\varepsilon^{a}(x) + f_{b}{}^{a}{}_{c}\varepsilon^{b}(x)A^{c}_{\mu}(x)$.

the proof) At first,①②③④ are assumed. ε_{0}^{r}=constant.

① $\psi_{A}(x) = \varepsilon^{r}(x)G_{rA}{}^{B}\psi_{B}(x)$.

② $D_{\mu}\psi_{A} = \partial_{\mu}\psi_{A} - A^{a}_{\mu}G_{rA}{}^{B}\psi_{B}$.

③ $0 = \delta\mathcal{L}(\psi_{A}, \partial_{\mu}\psi_{A}) = \varepsilon_{0}^{r}[\partial\mathcal{L}/\partial\psi_{A}\cdot G_{rA}{}^{B}\psi_{B} + \partial\mathcal{L}/\partial(\partial_{\mu}\psi)\cdot G_{rA}{}^{B}\partial_{\mu}\psi_{B}]$

④ $0 = \delta\mathcal{L}(\psi_{A}, D_{\mu}\psi_{A}) = \varepsilon_{0}^{r}[\partial\mathcal{L}/\partial\psi_{A}\cdot G_{rA}{}^{B}\psi_{B} + \partial\mathcal{L}/\partial(\partial_{\mu}\psi)\cdot G_{rA}{}^{B}D_{\mu}\psi_{B}]$

$0 = \delta\mathcal{L}(\psi_{A}, D_{\mu}\psi_{A}) = \delta\psi_{A}\,\partial\mathcal{L}/\partial\psi_{A} + \delta(D_{\mu}\psi)\,\partial\mathcal{L}/\partial(D_{\mu}\psi)$

$= \partial\mathcal{L}/\partial\psi_{A}\cdot\delta\psi_{A} + \partial\mathcal{L}/\partial(D_{\mu}\psi)\cdot[\partial_{\mu}\delta\psi_{A} - \delta A^{a}_{\mu}G_{aA}{}^{B}\psi_{B} - A^{a}_{\mu}G_{aA}{}^{B}\delta\psi_{B}]$

$= \partial\mathcal{L}/\partial\psi_{A}\cdot\delta\psi_{A} + \partial\mathcal{L}/\partial(D_{\mu}\psi)\cdot\partial_{\mu}\delta\psi_{A} - \partial\mathcal{L}/\partial(D_{\mu}\psi)\cdot[\delta A^{a}_{\mu}G_{aA}{}^{B}\psi_{B} + A^{a}_{\mu}G_{aA}{}^{B}\delta\psi_{B}]$

$= \varepsilon^{r}(x)\{\partial\mathcal{L}/\partial\psi_{A}\cdot G_{rA}{}^{B}\psi_{B} + \partial\mathcal{L}/\partial(D_{\mu}\psi_{A})\cdot G_{rA}{}^{B}(\partial_{\mu}\psi_{B} - A^{a}_{\mu}G_{aB}{}^{C}\psi_{C})\}$

$+ \varepsilon^{r}\,\partial\mathcal{L}/\partial(D_{\mu}\psi_{A})\cdot A^{a}_{\mu}G_{rA}{}^{B}G_{aB}{}^{C}\psi_{C} + \partial_{\mu}\varepsilon^{r}\,\partial\mathcal{L}/\partial(D_{\mu}\psi_{A})\cdot G_{rA}{}^{B}\psi_{B}$

$- \partial\mathcal{L}/\partial(D_{\mu}\psi)\cdot[\delta A^{a}_{\mu}G_{aA}{}^{B}\psi_{B} + \varepsilon^{r}A^{a}_{\mu}G_{aA}{}^{B}G_{rB}{}^{C}\psi_{C}]$

$= \partial_{\mu}\varepsilon^{r}\,\partial\mathcal{L}/\partial(D_{\mu}\psi_{A})\cdot G_{rA}{}^{B}\psi_{B} - \partial\mathcal{L}/\partial(D_{\mu}\psi)\cdot\delta A^{a}_{\mu}G_{aA}{}^{B}\psi_{B}$

$+ \varepsilon^{r}\,\partial\mathcal{L}/\partial(D_{\mu}\psi_{A})\cdot[A^{a}_{\mu}G_{rA}{}^{B}G_{aB}{}^{C}\psi_{C} - A^{a}_{\mu}G_{aA}{}^{B}G_{rB}{}^{C}\psi_{C}]$

$= \partial\mathcal{L}/\partial(D_{\mu}\psi_{A})\cdot[\partial_{\mu}\varepsilon^{r}\cdot G_{rA}{}^{B}\psi_{B} + \varepsilon^{r}A^{a}_{\mu}\cdot[G_{rA}{}^{B}G_{aB}{}^{C} - G_{aA}{}^{B}G_{rB}{}^{C}]\psi_{C} - \delta A^{a}_{\mu}G_{aA}{}^{B}\psi_{B}]$

$= \partial \mathcal{L}/\partial(D_\mu \psi_A)\cdot[\partial_\mu \varepsilon^a \cdot G_{aA}{}^B \psi_B + \varepsilon^b A^c{}_\mu \cdot f_b{}^a{}_c G_{aA}{}^B \psi_B - \delta A^a{}_\mu G_{aA}{}^B \psi_B]$

$= \partial \mathcal{L}/\partial(D_\mu \psi_A)\cdot[\partial_\mu \varepsilon^a + f_b{}^a{}_c \varepsilon^b A^c{}_\mu - \delta A^a{}_\mu] G_{aA}{}^B \psi_B$

Gauge Field Transform:

⑤ $\delta A^a{}_\mu(x) = \partial_\mu \varepsilon^a(x) + f_b{}^a{}_c \varepsilon^b(x) A^c{}_\mu(x) \equiv D_\mu \varepsilon^a(x)$.

[3] : Gauge Field Lagrangean.

⑥ $\mathcal{L}_{GF} = -\frac{1}{4}(\partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu - f_b{}^a{}_c A^b{}_\mu A^c{}_\nu)^2 \equiv -\frac{1}{4} F^a{}_{\mu\nu} F^a{}_{\mu\nu}$.

The conclusion is deriving above Lagrangean⑥ due to the **Gauge Field Transform**⑤.

(1) $0 = \delta \mathcal{L}_G(A^a{}_\mu; \partial_\nu A^a{}_\mu) = \delta A^a{}_\mu(\partial \mathcal{L}/\partial A^a{}_\mu) + \delta\, \partial_\nu A^a{}_\mu(\partial \mathcal{L}/\partial(\partial_\nu A^a{}_\mu))$.

$0 = [\partial_\mu \varepsilon^a + f_b{}^a{}_c \varepsilon^b A^c{}_\mu](\partial \mathcal{L}/\partial A^a{}_\mu)$

$+ (\partial \mathcal{L}/\partial(\partial_\nu A^a{}_\mu))[\partial_\nu \partial_\mu \varepsilon^a + f_b{}^a{}_c \partial_\nu \varepsilon^b A^c{}_\mu + f_b{}^a{}_c \varepsilon^b \partial_\nu A^c{}_\mu]$

$0 = \partial_\nu \partial_\mu \varepsilon^a \{\partial \mathcal{L}/\partial(\partial_\nu A^a{}_\mu)\} + \partial_\nu \varepsilon^b \{\partial \mathcal{L}/\partial A^b{}_\mu + f_b{}^a{}_c A^c{}_\mu(\partial \mathcal{L}/\partial(\partial_\nu A^a{}_\mu))\}$

$+ f_b{}^a{}_c \varepsilon^b \{A^c{}_\mu(\partial \mathcal{L}/\partial A^a{}_\mu) + \partial_\nu A^c{}_\mu(\partial \mathcal{L}/\partial(\partial_\nu A^a{}_\mu))\}$.

Those each {...........}must be zero.

(2) $0 = \partial_\nu \partial_\mu \varepsilon^a (\partial \mathcal{L}/\partial(\partial_\nu A^a{}_\mu))$

$= \partial_\nu \partial_\mu \varepsilon^a \{(\partial \mathcal{L}/\partial(\partial_\nu A^a{}_\mu)) + (\partial \mathcal{L}/\partial(\partial_\mu A^a{}_\nu)\}$

$0 = \partial \mathcal{L}/\partial(\partial_\nu A^a{}_\mu) + \partial \mathcal{L}/\partial(\partial_\mu A^a{}_\nu). \longrightarrow \mathcal{L}_G = \mathcal{L}_G(\partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + \ldots\ldots)$.

(3) $0 = \partial_\mu \varepsilon^a (\partial \mathcal{L}/\partial A^a{}_\mu) + f_b{}^a{}_c \partial_\nu \varepsilon^b A^c{}_\mu(\partial \mathcal{L}/\partial(\partial_\nu A^a{}_\mu))$

$= \partial_\mu \varepsilon^b (\partial \mathcal{L}/\partial A^b{}_\mu) + f_b{}^a{}_c \partial_\mu \varepsilon^b A^c{}_\nu (\partial \mathcal{L}/\partial(\partial_\mu A^a{}_\nu))$

$0 = \partial_\mu \varepsilon^b [(\partial \mathcal{L}/\partial A^b{}_\mu) + f_b{}^a{}_c A^c{}_\nu(\partial \mathcal{L}/\partial(\partial_\mu A^a{}_\nu))]$

⑥ $\mathcal{L}_{GF} = -\frac{1}{4} F^a{}_{\mu\nu} F^a{}_{\mu\nu} = -\frac{1}{4}(\partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu - f_b{}^a{}_c A^b{}_\mu A^c{}_\nu)^2$.

$\partial \mathcal{L}/\partial A^b{}_\mu = +\frac{1}{2}(\partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu - f_b{}^a{}_c A^b{}_\mu A^c{}_\nu) f_b{}^a{}_c A^c{}_\nu$.

$\partial \mathcal{L}/\partial(\partial_\mu A^a{}_\nu) = -\frac{1}{2}(\partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu - f_b{}^a{}_c A^b{}_\mu A^c{}_\nu)$.

$\rightarrow \quad [(\partial \mathcal{L}/\partial A^b{}_\mu) + f_b{}^a{}_c A^c{}_\nu(\partial \mathcal{L}/\partial(\partial_\mu A^a{}_\nu))] = 0$.

Therefore Lagrangean ⑥ has substantially been determined by (2)(3).

$\mathcal{L}_{GF}$ completely agrees with that of Quantum Electro Dyanamics($f_b{}^a{}_c = 0$).

(4) $0 = gf_b{}^a{}_c \varepsilon^b [A^c{}_\mu (\partial \mathscr{L} / \partial A^a{}_\mu) + \partial_\nu A^c{}_\mu (\partial \mathscr{L} / \partial (\partial_\nu A^a{}_\mu))]$。

This is awful relation which never be derived by direct calculation,but by following one.

This relation may not substantially concern for determining⑥. Here is Euler equation.

$0 = \partial \mathscr{L} / \partial A^a{}_\mu - \partial_\nu [\partial \mathscr{L} / \partial (\partial_\nu A^a{}_\mu)]$.

$0 = A^c{}_\mu \{\partial \mathscr{L} / \partial A^a{}_\mu - \partial_\nu [\partial \mathscr{L} / \partial (\partial_\nu A^a{}_\mu)]\}$

$= A^c{}_\mu \partial \mathscr{L} / \partial A^a{}_\mu + \partial_\nu A^c{}_\mu \partial \mathscr{L} / \partial (\partial_\nu A^a{}_\mu) - \partial_\nu [A^c{}_\mu \partial \mathscr{L} / \partial (\partial_\nu A^a{}_\mu)]$

$= A^c{}_\mu \partial \mathscr{L} / \partial A^a{}_\mu + \partial_\nu A^c{}_\mu \partial \mathscr{L} / \partial (\partial_\nu A^a{}_\mu)$.

☞: the gray term is vanished by the volume integral !!.

$* \ \partial_\nu [A^c{}_\mu \partial \mathscr{L} / \partial (\partial_\nu A^a{}_\mu)] = \partial_\nu A^c{}_\mu \partial \mathscr{L} / \partial (\partial_\nu A^a{}_\mu) + A^c{}_\mu \partial_\nu [\partial \mathscr{L} / \partial (\partial_\nu A^a{}_\mu)]$.

[4] : Quantized Lagrangean of ”B^a”<dipole field>.

⑥ $\mathscr{L}_{GF} = -\frac{1}{4} (\partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu - f_b{}^a{}_c A^b{}_\mu A^c{}_\nu)^2 \equiv -\frac{1}{4} F^a{}_{\mu\nu} F^a{}_{\mu\nu}$.

Note,in above **classical**(pre quantized)Lagragean⑥, $A^a{}_0$ has no **canonical conjugate momentum variable** $\Pi^a{}_0$.

$\Pi^a{}_{\mu=0} \equiv \partial \mathscr{L} / \partial (\partial_t A^a{}_0) = \partial \mathscr{L} / \partial ((ic)^{-1} \partial_0 A^a{}_0) = (ic) \partial \mathscr{L} / \partial (\partial_0 A^a{}_0) = 0$.

$* \ x_0 \equiv ict$.

The **Canonical Commutation Relation** must be as follows. Thereby $B(x_0,\mathbf{x})$ must be !!.

$[\mathbf{Q},\mathbf{P}] \equiv \mathbf{QP} - \mathbf{PQ} \equiv [B^a(x_0,\mathbf{x}), A^a{}_0(x_0,\mathbf{x}')] = i\hbar \delta(\mathbf{x} - \mathbf{x}')$.

*R.Utiyama:Prog Theor Phys. Suppl,9(1959),19-44.

As the consequence,quantized Lagrangean $\equiv \mathscr{L}_B$ is to emerge.

⑦ $\mathscr{L}_B \equiv (ic)^{-1} B^a \partial_\mu A^a{}_\mu + \frac{1}{2} \alpha^a B^a B^a$.

$\Pi^a{}_0 = (ic) \partial ((ic)^{-1} B^a \partial_0 A^a{}_0) / \partial (\partial_0 A^a{}_0) = B^a$.

However $(ic)^{-1} B^a \partial_0 A^a{}_0$ is not **symmetric in 4 dim space variable**,thereby it must be $(ic)^{-1} B^a \partial_\mu A^a{}_\mu$. $+\frac{1}{2} \alpha^a B^a B^a$ is due to **the Euler Equation**.

⑧ $0 = \partial \mathscr{L}_B / \partial B^a = (ic)^{-1} \partial_\mu A^a{}_\mu + \alpha^a B^a$. <$\alpha = -1/\varepsilon_0$ in Quantum Electrodynamics=QED>

Note someone once told “α” is not indefinite,but the fact is definite. Also note B^a has dimension of **dipole density** in QED.That is, B^a is not matter,but vacuum field<=gohst>.

☞: In following [5],we discuss gauge covariant on $\mathscr{L}_B(B^a, \partial_\mu A^a{}_\mu)$ by $\delta A^a{}_\mu$, but $\delta B^a = 0$. Because ***gauge must be observable*** in Quantum Dynamics Concept,while the gohst B^a is non observable due to <phys|B^a|phys>$=0$ in classical meaning.

*C.Becchi,A.Rouet,and R.Stora:Comm Math Phys,**42**(1975)127.,, :Ann.Phys.**98**(1976)287.

[5]: Quantized Lagrangean of $\{\bar{C}^a . C^a\}$<FP Gohst>.

*L.D.Faddeev and V.N.Popov:Pphys Lett.25B(1967)29.

APPENDIX3:Deriving FP Lagrangean by Path-Integral.

The Euler Equation ⑧ must be gauge invariant by $\delta A^a{}_\mu$.

$$0 = ic\,\partial_\mu A^a{}_\mu + \alpha B^a = ic\,\partial_\mu (A^a{}_\mu + \delta A^a{}_\mu) + \alpha B^a = ic\,\partial_\mu \delta A^a{}_\mu = ic\,\partial_\mu D_\mu \varepsilon^a = 0 \quad \ldots\ldots ⑨'$$

$$⑨ \quad 0 = ic\,\partial_\mu D_\mu C^a = ic[\partial_\mu \partial_\mu C^a + f_b{}^a{}_c \partial_\mu C^b A^c{}_\mu + f_b{}^a{}_c C^b \partial_\mu A^c{}_\mu).$$

(1)**Being of another ghost field<Faddev-Popov gohst>.**

Thus gauge invariant is to yield another constrain equation ⑨' on $\{\varepsilon^a(x)\}$.Then remember **[1]**(3) → $\partial_\mu \varepsilon^a(x) \equiv A^a{}_\mu(x)$. This should be called the 1st constrain equation ,while ⑨ should be called the 2nd constrain equation on $\{\varepsilon^a(x)\}$.It must be another field $\{C^a(x)\}$<**FP gohst**>.Thereby we must conclude the field equation ⑨, the meaning of which is decisive !.

(2)**Analogy between Spinor interaction** $\{\bar{\phi}\ ;\ \phi\}$ with $A^a{}_\mu$.

Especially note that $\partial_\mu C^a$ is to interact with gauge field $A^a{}_\mu$ in ⑨. Then there must be **Noether current** by $\{\partial_\mu \bar{C}^a ; C^a\}$ similar with spinor Noether current $\{\bar{\phi}\ ;\ \phi\}$ by $\{$**[1]**(4)(b)$\}$.

$$\mathcal{L}_S(\phi, A^a{}_\mu) = -c\,\bar{\phi}(\hbar\gamma^\mu D_\mu + mc)\phi = -c\,\bar{\phi}(\hbar\gamma^\mu(\partial_\mu - A^a{}_\mu(x)G_a) + mc)\phi .$$

[1](4)(b)Euler Equation: $0 = \partial\mathcal{L}_S / \partial\bar{\phi} = -c(\hbar\gamma^\mu(\partial_\mu - A^a{}_\mu(x)G_a) + mc)\phi$.

The **Noether current** $j^a{}_\mu(x) \equiv -c\hbar\,\bar{\phi}\gamma^\mu G_a \phi$.

(3)**FP Lagrangean** $\equiv \mathcal{L}_{FP}$.

$$⑩ \quad \mathcal{L}_{FP}(\bar{C}^a(x), C^a) \equiv \chi\,\bar{C}^a \cdot \partial_\mu D_\mu C^a = \chi\,\partial_\mu[\bar{C}^a \cdot \partial_\mu D_\mu C^a] - \chi\,\partial_\mu \bar{C}^a \cdot D_\mu C^a = -\chi\,\partial_\mu \bar{C}^a \cdot D_\mu C^a .$$

$$= -\chi\,\partial_\mu \bar{C}^a \cdot \partial_\mu C^a - \chi\,\partial_\mu \bar{C}^c \cdot f_b{}^c{}_a C^b A^a{}_\mu]. \rightarrow j_{FP}{}^a{}_\mu(x) \equiv \chi\,\partial_\mu \bar{C}^c \cdot f_b{}^a{}_c C^b .$$

Euler Equation-1: $0 = \partial\mathcal{L}_{FP} / \partial\bar{C}^a = \chi\partial_\mu D_\mu C^a = 0$.

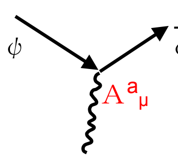
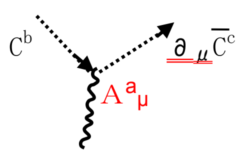
Euler Equation-2: $\mathcal{L}_{FP}(\bar{C}^a(x), C^a) = -\chi\partial_\mu \bar{C}^a \cdot \partial_\mu C^a - C^a f_a{}^b{}_c \partial_\mu \bar{C}^b A^c{}_\mu$.

$$0 = \partial\mathcal{L}/\partial C^a - \partial_\mu[\partial\mathcal{L}/\partial(\partial_\mu C^a)] = \chi(\partial_\mu\partial_\mu \bar{C}^a - gf_a{}^b{}_c \partial_\mu \bar{C}^b A^c{}_\mu) = \chi D_\mu \partial_\mu \bar{C}^a = 0.$$

(4)**Noether Current Interaction** $= j^a{}_\mu A^a{}_\mu$ in **FP Gohst Lagragean.**

$*\ \mathcal{L}_{FPI}(\bar{C}^a, C^a) = -\chi f_b{}^a{}_c \partial_\mu \bar{C}^b \cdot C^c \cdot A^a{}_\mu$.

$*\ \mathcal{L}_I(\bar{\phi}, \phi) = -gc\hbar\,\bar{\phi}\gamma^\mu A^a{}_\mu(x) G_a \phi$ …… $j^a{}_\mu A^a{}_\mu$ type interaction

$\mathcal{L}_I = -gc\hbar\,\bar{\phi}\gamma^\mu A^a{}_\mu(x) G_a \phi$.	$\mathcal{L}_{FPI} = \chi g\,\partial_\mu \bar{C}^a f_b{}^a{}_c C^b A^c{}_\mu$.
	

In Feymann diagram,
We see **symmetry of matter & vaccum field**

APPENDIX-1:

In this report,authors try to enable non-expert's understanding.However readers are assumed to have once learned Quantum Physics.Thereby,here are technical supplement.

(1)Surface Integral Vanishing(an example) in deriving Euler Equation.

$0=\delta\mathcal{L}(A^a_\mu;\partial_\nu A^a_\mu)=\delta A^a_\mu\cdot\partial\mathcal{L}/\partial A^a_\mu+\delta\partial_\nu A^a_\mu\cdot\partial\mathcal{L}/\partial(\partial_\nu A^a_\mu)$

$=\delta A^a_\mu\cdot\partial\mathcal{L}/\partial A^a_\mu+\partial_\nu[\delta A^a_\mu\cdot\partial\mathcal{L}/\partial(\partial_\nu A^a_\mu)]-\delta A^a_\mu\cdot\partial_\nu[\partial\mathcal{L}/\partial(\partial_\nu A^a_\mu)]$

$=\delta A^a_\mu\cdot[\partial\mathcal{L}/\partial A^a_\mu-\partial_\nu[\partial\mathcal{L}/\partial(\partial_\nu A^a_\mu)]]+\partial_\nu[\delta A^a_\mu\cdot\partial\mathcal{L}/\partial(\partial_\nu A^a_\mu)]$

$*\delta\partial_\nu A^a_\mu=\partial_\nu\delta A^a_\mu\cdot$ δA^a_μ is arbitrary.

*$\mathcal{L}$ is originally defined as volume density dimension,

so it must be redefined by time and space integral.

$\int dx_0 \oiint dx^3\,\partial_\nu[\delta A^a_\mu\cdot\partial\mathcal{L}/\partial(\partial_\nu A^a_\mu)]$

$=\oiint dx^3\{\int dx_0\,\partial_0[\delta A^a_\mu\cdot\partial\mathcal{L}/\partial(\partial_\nu A^a_\mu)]+\int dx_0\oiint dx^3\,\partial_k[\delta A^a_k\cdot\partial\mathcal{L}/\partial(\partial_\nu A^a_k)]\}$

The 1st term could be zero by taking $\delta A^a_\mu=0$ at initial and final time,while 2nd term becomes surface integral at infinity where **inte-grand** becomes zero.

(2)As for FP Lagrangean Deriving.

Original Poof needs knowledge on path integral and others.The method at here is a simple way by authors.However **the conclusion is correct**.

(3)B has dipole density dimension.<Q=charge,L=length,t=time,M=mass>

$[j_kA_k]$=energy density$=L\cdot MLt^{-2}L^{-3}$

$[j_k]$=current density$=Q\cdot Lt^{-1}L^{-3}$

$[A_k]=[j_kA_k]/[j_k]=L\cdot MLt^{-2}L^{-3}/Q\cdot Lt^{-1}L^{-3}=MLt^{-1}Q^{-1}$

δ function $[\hbar]=[BA_k]$=energy$\cdot$time$\cdot L^{-3}=t\cdot L\cdot MLt^{-2}$

$[B]=[\hbar]/[A_k]=L^{-3}t\cdot L\cdot MLt^{-2}/MLt^{-1}Q^{-1}=(QL)L^{-3}$

(4) $\alpha=-1/\varepsilon_0$,

This is not zero,where ε_0 is permeability.

http://www.777true.net/QED1.pdf

APPENDIX2:Gravity Field as Complete Gauge One

(for establishing Quantum Gravity Dynamics=QGD).

[1]: Principle of Equivalence=Local Lorentz Covariance.

Local Lorentz Covariance=Local Gauge Covariance.

Equivalent Principle(A.Einstein 1917).
Uniformly accelerated every kind of particles motion by constant gravity is completely vanished(free particles-nization) by observing by accelerated coordinates.That is, gravity field and accelerated coordinate is equivalent.

Local Lorentz Covariance(R.Utiyama 1956).
Global Lorentz Transform is defined as transform between coordinates with different,but constant velocity,This is free field.While **accelerated particle system with interaction**,only local space & spot,Lorentz transform is applicable(**Local Lorentz Transform**(R.Utiyama 1956)). This is nothing,but complete mathematical representation of **Equivalent Principle** (A.Einstein 1917).Note general theory of relativity could not be quantized,while this method is not !,that is,**general relativity theory** by general curve-linear coordinates is not exact in quantum meaning,but may be macroscopical approximation.Now we show proof that **Local Lorentz Covariant** can becomes simultaneously **Local Gauge Covariant** completely.

[2]: Local Lorentz Covariance by (local)Linear Coordinates.

Gauge transform is only on inner coordinate$\{\phi ; D_\mu \phi\}$.While this is **both transform** on cordinate(x_μ) and inner one$\{\phi ; D_\mu \phi\}$.Even though,this is to become gauge one !!.

Our aim is deriving covariant gauge field transform as follows by Local Lorentz transform.
$\delta A^{kl}{}_{\mu}(x) = \partial_\mu \varepsilon^{kl}(x) + \frac{1}{4}\varepsilon^{mn}(x) A^{op}{}_{\mu}(x) f_{mn}{}^{kl}{}_{op}$.

(1)**local Lorentz transform of time space coordinate in local linear coordinate:**

$*\ \varepsilon^{\mu}{}_{\nu}(x)$ is infinitesimal parameter depending on($x_\mu=[x_0=ict,x_1,x_2,x_3]$).

$dx'^{\mu} = a^{\mu}{}_{\nu} dx^{\nu} = (\delta^{\mu}{}_{\nu} + \varepsilon^{\mu}{}_{\nu}(x))dx^{\nu}$. $\rightarrow$ $dx'^{\mu} dx'^{\mu} = dx^{\nu} dx^{\nu}$. $\rightarrow$ $\varepsilon^{\mu}{}_{\nu}(x) = -\varepsilon^{\nu}{}_{\mu}(x)$.

$*$differential operator : $\partial'_{\mu} = (\partial x^{\nu} / \partial x'^{\mu})\partial_{\nu} = a^{-1\nu}{}_{\mu}\partial_{\nu}$.

(2)**spinor field transform:**<☞:**note** spinor is defined only by **linear coordinates!!!**>

$\phi'_{A}(x') = T_A{}^{B}\phi_{B}(x)$; <$T = (1 + \frac{1}{4}\varepsilon_{\alpha\beta}(x)\gamma^{\alpha}\gamma^{\beta}) \equiv (1 + \frac{1}{2}\varepsilon^{kl}(x) G_{kl})$>.

<$\gamma^{\alpha}\gamma^{\beta} + \gamma^{\beta}\gamma^{\alpha} = 2\delta^{\alpha\beta}$; $G_{kl} \equiv \frac{1}{2}\gamma^{k}\gamma^{l}$>.

$\bar{\phi}'(x') = (T\phi(x))^{\ddagger}\gamma^{0} = \bar{\phi}(x)T^{-1}$. < $\ddagger$ =take complex conjugate=* and transpose matrix=t>

(3)global Lorentz covariant Lagrangean< $\varepsilon_{\alpha\beta}$=constant>.

$\mathcal{L}'(x') \equiv -c\bar{\phi}'(x')(\hbar\gamma^{\mu}\partial'_{\mu} + mc)\phi'(x') = -c\bar{\phi}(x)T^{-1}(\hbar\gamma^{\mu}a^{-1\nu}{}_{\mu}\partial_{\nu} + mc)T\phi(x)$

$= -c\bar{\phi}(x)(\hbar\langle T^{-1}\gamma^{\mu}a^{-1\nu}{}_{\mu}T\rangle\partial_{\nu} + mc)\phi(x) \equiv -c\bar{\phi}(x)(\hbar\gamma^{\nu}\partial_{\nu} + mc)\phi(x)$.

$\rightarrow$ $T^{-1}\gamma^{\mu}a^{-1\nu}{}_{\mu}T = \gamma^{\nu}$..........(3)

(4)$\mathcal{L}(\phi_A(x);D_\mu\phi_A(x))$:**Spinor Lagrangean local Covariance**:

$\mathcal{L}'(x') \equiv -c\bar{\phi}'(x')(\hbar\gamma^\mu(\partial'_\mu - \frac{1}{2}A'^{kl}_\mu(x')G_{kl}) + mc)\phi'(x')$ ← this is gauge covariant form

$= -c\bar{\phi}(x)T^{-1}(\hbar\gamma^\mu(a^{-1\nu}{}_\mu\partial_\nu - \frac{1}{2}A'^{kl}_\mu(x')G_{kl}) + mc)T\phi(x)$

$= -c\bar{\phi}[\hbar T^{-1}\gamma^\mu a^{-1\nu}{}_\mu\partial_\nu - \frac{1}{2}\hbar T^{-1}\gamma^\mu A'^{kl}_\mu(x')G_{kl} + mcT^{-1}]T\phi$

$= -c\bar{\phi}[\hbar T^{-1}\hbar\gamma^\mu a^{-1\nu}{}_\mu\partial_\nu(T\phi) - \frac{1}{2}\hbar T^{-1}\gamma^\mu A'^{kl}_\mu(x')G_{kl}T\phi + mc\phi]$

$= -\bar{c}\phi[\hbar T^{-1}\gamma^\mu a^{-1\nu}{}_\mu T\partial_\nu - \frac{1}{2}\hbar\gamma^\mu A^{kl}_\mu G_{kl} + mc]\phi(x)$

$-c\bar{\phi}[\hbar T^{-1}\hbar\gamma^\mu a^{-1\nu}{}_\mu\partial_\nu Tx + \frac{1}{2}\hbar\gamma^\mu A^{kl}_\mu G_{kl} - \frac{1}{2}\hbar T^{-1}\gamma^\mu A'^{kl}_\mu(x')G_{kl}T]\phi$

$= \mathcal{L}(x) - c\hbar\bar{\phi}[(T^{-1}\gamma^\mu a^{-1\nu}{}_\mu T)T^{-1}\partial_\nu Tx + \frac{1}{2}\gamma^\mu A^{kl}_\mu G_{kl} - \frac{1}{2}T^{-1}\gamma^\mu A'^{kl}_\mu(x')G_{kl}T]\phi$

$= \mathcal{L}(x) - c\hbar\bar{\phi}[(T^{-1}\gamma^\mu a^{-1\nu}{}_\mu T)T^{-1}\partial_\nu Tx + \frac{1}{2}\gamma^\mu A^{kl}_\mu G_{kl} - \frac{1}{2}T^{-1}\gamma^\mu A'^{kl}_\mu(x')G_{kl}T]\phi$ ←(3)

$= \mathcal{L}(x) - c\hbar\bar{\phi}[\gamma^\nu T^{-1}\partial_\nu Tx + \frac{1}{2}\gamma^\mu A^{kl}_\mu G_{kl} - \frac{1}{2}T^{-1}\gamma^\mu A'^{kl}_\mu(x')G_{kl}T]\phi$.

[The second term] must be vanished,so we derive following relations.

$\frac{1}{2}T^{-1}\gamma^\mu A'^{kl}_\mu(x')G_{kl}T = \frac{1}{2}\gamma^\mu A^{kl}_\mu G_{kl} - \gamma^\nu T^{-1}\partial_\nu T$.

$\frac{1}{2}\gamma^\mu A'^{kl}_\mu(x')G_{kl} = T\frac{1}{2}\gamma^\mu A^{kl}_\mu G_{kl}T^{-1} - T\gamma^\nu T^{-1}\partial_\nu TT^{-1}$.

$*0 = \partial_\nu(TT^{-1}) = \partial_\nu TT^{-1} + T\partial_\nu T^{-1}$..

Transform Rule-1 in Gravitational Gauge Field.

(4) $\frac{1}{2}\gamma^\mu A'^{kl}_\mu(x')G_{kl} = \frac{1}{2}T\gamma^\mu A^{kl}_\mu G_{kl}T^{-1} - T\gamma^\nu\partial_\nu T^{-1}$..

(5)**Deriving Gauge Field Transform by Local Lorentz Transform**.

Transform Rule-2 in Gravitational Gauge Field.

(5) $\delta A^{kl}_\mu(x) = \partial_\mu\varepsilon^{kl} + \frac{1}{4}\varepsilon^{mn}A^{op}_\mu f_{mn}{}^{kl}{}_{op}$.

$*\partial_\mu\varepsilon^{kl}(x) = A^{kl}_\mu(x)$.

$*D_\mu\phi(x) \equiv \lim\ \Delta x \to 0[\phi(x+\Delta x) - \phi(x+\Delta x)_{//}]/\Delta x = \partial_\mu\phi(x) - A^a_\mu(x)G_a\phi(x)$.

$*T = (1 - \frac{1}{2}\varepsilon^{kl}G_{kl})$; $T = (1 + \frac{1}{2}\varepsilon^{kl}G_{kl})$, note sign of ε^{kl} is changed at first here.

$\frac{1}{2}\gamma^\mu\delta A^{kl}_\mu(x)G_{kl} \equiv \frac{1}{2}\gamma^\mu[A'^{kl}_\mu(x') - A^{kl}_\mu(x)]G_{kl}$

$= \frac{1}{2}T\gamma^\mu A^{kl}_\mu G_{kl}T^{-1} - T\gamma^\nu\partial_\nu T^{-1} - \frac{1}{2}\gamma^\mu A^{kl}_\mu(x)G_{kl}$

$= \frac{1}{2}(1 - \frac{1}{2}\varepsilon^{kl}G_{kl})\gamma^\mu A^{mn}_\mu G_{mn}(1 + \frac{1}{2}\varepsilon^{op}G_{op}) - (1 - \frac{1}{2}\varepsilon^{kl}G_{kl})\gamma^\nu\partial_\nu(1 + \frac{1}{2}\varepsilon^{mn}G_{mn})$

$-\frac{1}{2}\gamma^\mu A^{kl}_\mu(x)G_{kl}$

$= -\frac{1}{4}\varepsilon^{kl}G_{kl})\gamma^\mu A^{mn}_\mu G_{mn} + \frac{1}{4}\gamma^\mu A^{mn}_\mu G_{mn}\varepsilon^{op}G_{op} - \frac{1}{2}\gamma^\nu\partial_\nu\varepsilon^{mn}G_{mn} + \frac{1}{4}\varepsilon^{kl}G_{kl}\gamma^\nu A^{mn}_\nu G_{mn}$

$= -\frac{1}{4}\varepsilon^{kl}A^{mn}_\mu G_{kl}\gamma^\mu G_{mn} + \frac{1}{4}\varepsilon^{op}A^{mn}_\mu\gamma^\mu G_{mn}G_{op} - \frac{1}{2}\partial_\nu\varepsilon^{mn}\gamma^\nu G_{mn} + \frac{1}{4}\varepsilon^{kl}A^{mn}_\nu G_{kl}\gamma^\nu G_{mn}$

$=\frac{1}{4}\gamma^{\mu}\varepsilon^{op}A^{mn}{}_{\mu}G_{mn}G_{op}-\frac{1}{2}\gamma^{\nu}\partial_{\nu}\varepsilon^{mn}G_{mn}$

$=\frac{1}{4}(\varepsilon^{op}\partial_{\mu}\varepsilon^{mn})\gamma^{\mu}G_{mn}G_{op}-\frac{1}{2}\gamma^{\mu}\partial_{\mu}\varepsilon^{kl}G_{kl}$

$=\frac{1}{4}(\partial_{\mu}\langle\varepsilon^{op}\varepsilon^{mn}\rangle-\partial_{\mu}\varepsilon^{op}\varepsilon^{mn})\gamma^{\mu}G_{mn}G_{op}-\frac{1}{2}\gamma^{\mu}\partial_{\mu}\varepsilon^{kl}G_{kl}$

$=\frac{1}{4}(-\partial_{\mu}\varepsilon^{op}\varepsilon^{mn})\gamma^{\mu}G_{mn}G_{op}-\frac{1}{2}\gamma^{\mu}\partial_{\mu}\varepsilon^{kl}G_{kl}$

$=-\frac{1}{4}\gamma^{\mu}A^{op}{}_{\mu}\varepsilon^{mn}G_{mn}G_{op}-\frac{1}{2}\gamma^{\mu}\partial_{\mu}\varepsilon^{kl}G_{kl}$

$=-\frac{1}{4}\gamma^{\mu}A^{mn}{}_{\mu}\varepsilon^{op}G_{op}G_{mn}-\frac{1}{2}\gamma^{\mu}\partial_{\mu}\varepsilon^{kl}G_{kl}$

$=\frac{1}{2}\{.....+....\}$

$=\frac{1}{2}\gamma^{\mu}[\frac{1}{4}\varepsilon^{op}A^{mn}{}_{\mu}[G_{mn}G_{op}-G_{op}G_{mn}]-\frac{1}{2}\partial_{\mu}\varepsilon^{kl}G_{kl}]$

$=\frac{1}{2}\gamma^{\mu}[\frac{1}{4}\varepsilon^{op}A^{mn}{}_{\mu}f_{mn}{}^{kl}{}_{op}G_{kl}-\partial_{\mu}\varepsilon^{kl}G_{kl}]$

$=\frac{1}{2}\gamma^{\mu}[-\frac{1}{4}\varepsilon^{mn}A^{op}{}_{\mu}f_{mn}{}^{kl}{}_{op}-\partial_{\mu}\varepsilon^{kl}]G_{kl}$

$\delta A^{kl}{}_{\mu}(x)=-\partial_{\mu}\varepsilon^{kl}-\frac{1}{4}\varepsilon^{mn}A^{op}{}_{\mu}f_{mn}{}^{kl}{}_{op}$. Let's remind sign change !,

$\rightarrow\quad \delta A^{kl}{}_{\mu}(x)=\partial_{\mu}\varepsilon^{kl}+\frac{1}{4}\varepsilon^{mn}A^{op}{}_{\mu}f_{mn}{}^{kl}{}_{op}$. <proof end>

(6)**QGD Gauge Field Lagrangean**.

$F^{kl}{}_{\mu}(x)=(\partial_{\mu}A^{kl}{}_{\nu}-\partial_{\nu}A^{kl}{}_{\mu}-\frac{1}{4}f_{mn}{}^{kl}{}_{op}A^{mn}{}_{\mu}A^{op}{}_{\nu})$.

$\mathcal{L}_{A}=-(1/4\eta)(\partial_{\mu}A^{kl}{}_{\nu}-\partial_{\nu}A^{kl}{}_{\mu}-\frac{1}{4}f_{mn}{}^{kl}{}_{op}A^{mn}{}_{\mu}A^{op}{}_{\nu})^{2}$.

(7)**QGD Lagrangean**. $\langle\mu,\nu,k,l,...=0,1,2,3,.....,11\rangle$

$\mathcal{L}_{QGD}(\phi,D_{\mu}\phi;B^{kl};C^{kl})$

$=-c\overline{\phi}(\hbar\gamma^{\mu}(\partial_{\mu}-\frac{1}{2}A^{kl}{}_{\mu}(x)G_{kl})+mc)\phi$.

$-(1/4\eta)(\partial_{\mu}A^{kl}{}_{\nu}-\partial_{\nu}A^{kl}{}_{\mu}-\frac{1}{4}f_{mn}{}^{kl}{}_{op}A^{mn}{}_{\mu}A^{op}{}_{\nu})^{2}$

$+(ic/2)\partial_{\mu}A^{kl}{}_{\mu}B^{kl}+(\alpha^{kl}/4)B^{kl}B^{kl}+(\chi/2)\partial_{\mu}\overline{C}^{kl}D_{\mu}C^{kl}$.

(8)**Notable Singularity of QGD in general gauge theory**.

$(x_{\mu}=[x_0=ict,x_1,x_2,x_3,....,x_{11}])$.

$dx'^{0}=\varepsilon^{0}{}_{\nu\neq0}(x)dx^{\nu}$. $\rightarrow$ d(ict')=imaginary,while $dx^{\nu\neq0}$=real,thereby,.

$\varepsilon^{0k}(x)$=imaginary, $A^{0k}{}_{\mu>0}(x)=\partial_{\mu\neq0}\varepsilon^{0k}(x)$=imaginary,

In QGD,there exist anti-hermite gauge field $A^{0k}{}_{\mu>0}(x)$. <transversal components>.

Anti-hermite means non physical(,but virtual someone !!,who could do creation form 0?!).

This components is to have initial over negative energy toward creating positive energy to satisfy **energy conservation law**: $0=+E-E$ in creation of this universe(**BigBang**).

Process time Δt is short enough by uncertain principle $\Delta E\,\Delta t=\hbar$. <Plank constant>,

where $\Delta E=+E-E<0$ is breaking down amount of energy conservation law.

Note also creating business is initiated by debt !.

http://www.777true.net/Energy-Creation-Process-from-QED-to-QGD.pdf

http://www.777true.net/img008-Quick-Guide-to-Quantum-Gravitational-Dynamics.pdf

Also note debt is to terminate business, who neglect 0=total bond－total debt !!.

APPENDIX3:Deriving FP Lagrangean by Path-Integral. 2017/4/10

Gauge Covariant Quantized Lagrangean must be with $0=ic\partial_\mu D_\mu \varepsilon^a$,which is kernel.

Feyman Path Integral, Variable transform and Jacobian in integral calculation are tools.

Consequently,we derive Faddev-Popov Lagrangean term in **Gauge Field Quantization**.

*This is alternative of **[5]: Quantized Lagrangean of {$\bar{C}^a$.C^a}<FP Gohst>.**

*L.D.Faddeev and V.N.Popov:Pphys Lett.25B(1967)29.

"Feynman Diagram for The Yang-Mills Field"

Feynman, R. P. (1948). *Reviews of Modern Physics*. **20 (2): 367–387.

"Space-Time Approach to Non-Relativistic Quantum Mechanics".

[1]: Schrödinger EQN Solution by Path-Integral.

(1)$i\hbar\partial_t\Psi(t)=\mathbf{H}(t)\Psi(t)$. → $\Psi(t+\Delta t)=[1+\Delta t/i\hbar)\mathbf{H}(t)]\Psi(t)$

Difficulty of time & energy variable by uncertainty principle(UP) in Quantum Mechanics.

$\mathbf{H}(t)$ is energy observable,while (t) is time,which are ruled by UP($\Delta E\Delta t=\hbar$).Thereby,both can not be determined simultaneously without 0 error.Discussion at here is to neglect the fact(classical calculation),so It is inevitable to face **some difficulty** to derive definite result.

http://www.777true.net/img007-Quick-Guide-to-Quantum-Stochastic-Mechanics.pdf

☞:time at here is mere events sequence parameters $t_j>t_{j-1}$,but not time value.

(2)$\Psi(t_0+n\Delta t=t)=[1+(\Delta t/i\hbar)\mathbf{H}(t_0+\langle n-1\rangle\Delta t)]\Psi(t_0+\langle n-1\rangle\Delta t))$

$\Psi(t_0+\langle n-1\rangle\Delta t))=[1+(\Delta t/i\hbar)\mathbf{H}(t_0+\langle n-2\rangle\Delta t)]\Psi(t_0+\langle n-2\rangle\Delta t))$

.... $\Psi(t_0+\Delta t)=[1+(\Delta t/i\hbar)\mathbf{H}(t+0\Delta t)]\Psi(t_0+0.\ \Delta t=t_0))$.

$\Psi(t)=n\to\infty[1+(\Delta t/i\hbar)\mathbf{H}(t_{n-1})]\times[1+(\Delta t/i\hbar)\mathbf{H}(t_{n-2})]\times....\times[1+(\Delta t/i\hbar)\mathbf{H}(t_j)]\times$

$\times[1+(\Delta t/i\hbar)\mathbf{H}(t_1)]\times[1+(\Delta t/i\hbar)\mathbf{H}(t_0)]\Psi(t_0)\equiv\mathbf{S}(t;t_0)\Psi(t_0)$

$R_{fi}\equiv\langle\Psi(t)|\mathbf{S}(t;t_0)|\Psi(t_0)\rangle\equiv$transition probability amplitude from $\Psi(t_0)\to\Psi(t)$.

(3)Representation by $(\mathbf{Q};\mathbf{P})$ Space and Momentum Observable's Eingen Function Set.

(a)$\mathbf{P}|p\rangle=-i\hbar\partial_q|\exp(-pq/i\hbar)\rangle/\sqrt{(2\pi\hbar)}\rangle$.

$\langle p'|p\rangle=\int_{-\infty}^{\infty}dq\exp(p'q/i\hbar)\exp(-pq/i\hbar)/(2\pi\hbar)=\delta(p-p')$.

(b)$\mathbf{Q}|q\rangle=q'\delta(q-q')$.

(c)$\langle q|p\rangle\equiv\int dq'\delta(q-q')\exp(-pq'/i\hbar)\rangle/\sqrt{(2\pi\hbar)}\rangle\equiv\exp(-pq/i\hbar)\rangle/\sqrt{(2\pi\hbar)}\rangle$.

$\langle p|q\rangle\equiv\int_{-\infty}^{\infty}dq'\delta(q-q')\exp(pq'/i\hbar)\rangle/\sqrt{(2\pi\hbar)}\rangle\equiv\exp(pq/i\hbar)\rangle/\sqrt{(2\pi\hbar)}\rangle$.

(d)**unit operator** $1\equiv\int dq|q\rangle\langle q|=\int dq|p\rangle\langle p|$;

(4)**QP representation of** $S(t;t_0)$.

$S(t;t_0) = \int dq_{n-1}|q_{n-1}\rangle\langle q_{n-1}|[1+(\Delta t/i\hbar)\mathbf{H}(t_{n-1})]\int dp_{n-1}|p_{n-1}\rangle\langle p_{n-1}|$

$\times\int dq_{n-2}|q_{n-2}\rangle\langle q_{n-2}|[1+(\Delta t/i\hbar)\mathbf{H}(t_{n-2})]\int dp_{n-2}|p_{n-2}\rangle\langle p_{n-2}|\times$

....

$\times\int dq_j|q_j\rangle\langle q_j|[1+(\Delta t/i\hbar)\mathbf{H}(t_j)]\int dp_j|p_j\rangle\langle p_j|\times$

......

$\times\int dq_1|q_1\rangle\langle q_1|[1+(\Delta t/i\hbar)\mathbf{H}(t_1)]\int dP_1|p_1\rangle\langle p_1|$

$\times\int dq_0|q_0\rangle\langle q_0|[1+(\Delta t/i\hbar)\mathbf{H}(t_0)]\int dp_0|p_0\rangle\langle p_0|$

$\times\int dq_{-1}|q_{-1}\rangle\langle q_{-1}|$

$S(t;t_0) = \Pi_{j=-1}^{n-1}\int dq_j\Pi_{j=0}^{n-1}\int dp_j|q_{n-1}\rangle\langle q_{-1}|$

$\times\{\Pi_{j=0}^{n-1}\langle q_j|[1+(\Delta t/i\hbar)\mathbf{H}(t_j)]|p_j\rangle\langle p_j|q_{j-1}\rangle\}$

$*\langle q_j|[1+\Delta t/i\hbar)\mathbf{H}(t_j)]|p_j\rangle\langle p_j|q_{j-1}\rangle = [\langle q_j|p_j\rangle+\Delta t/i\hbar)\langle q_j|\mathbf{H}(t_j)|p_j\rangle]\langle p_j|q_{j-1}\rangle$

$=[\exp(-q_jp_j/i\hbar)\rangle/\sqrt{}(2\pi\hbar)+(\Delta t/i\hbar)\langle q_j|\mathbf{H}(t_j)|p_j\rangle]\exp(p_jq_{j-1}/i\hbar)\rangle/\sqrt{}(2\pi\hbar)$

$=\exp(-p_j\langle q_j-q_{j-1}\rangle/i\hbar)/(2\pi\hbar)+(\Delta t/i\hbar)\langle q_j|\mathbf{H}(t_j)|p_j\rangle]\exp(p_jq_{j-1}/i\hbar)\rangle/\sqrt{}(2\pi\hbar)$

$=\exp(-p_j\langle q_j-q_{j-1}\rangle/i\hbar)/(2\pi\hbar)[1+(\Delta t/i\hbar)\langle q_j|\mathbf{H}(t_j)|p_j\rangle]\exp(p_jq_j/i\hbar)\rangle\sqrt{}(2\pi\hbar)]$

* <usefurl formula : $1+\delta X=\exp(\delta X)$>

$*\exp(-p_j\langle q_j-q_{j-1}\rangle/i\hbar)=\exp(-\Delta t.\,p_j(dq_j/dt)/i\hbar)$.

$**|p_j'\rangle=\exp(-p_j'q_j/i\hbar)/\sqrt{}(2\pi\hbar).\to\langle p'|p\rangle=\delta(p-p')$.

$\langle q_j|\mathbf{H}(j)|p_j\rangle\sqrt{}(2\pi\hbar)\exp(p_jq_j/i\hbar)=\int dq_j'\mathbf{H}(q_j;p_j)\delta(q_j-q'_j)\exp(p_jq_j/i\hbar)\exp(-p_jq'_j/i\hbar)$

$=\mathbf{H}(q_j;p_j)$.

$=(2\pi\hbar)^{-1}.\exp[-(\Delta t//i\hbar).\,p_j(dq_j/dt)]\exp[(\Delta t/i\hbar).\,\mathbf{H}(q_j;p_j)]$

$=(2\pi\hbar)^{-1}.\exp[-(\Delta t/i\hbar)\langle\mathcal{L}(q_j;dq_j/dt)\rangle]$. <usefurl formular : $1+\delta X=\exp(\delta X)$>

$S(t;t_0)=\Pi_{j=-1}^{n-1}\int dq_j\Pi_{j=0}^{n-1}\int dp_j|q_{n-1}\rangle\langle q_{-1}|$

$\times\{\Pi_{j=0}^{n-1}(2\pi\hbar)^{-n}.\exp[-(\Delta t/i\hbar)\langle\mathcal{L}(q_j;dq_j/dt)\rangle]$.

$=\Pi_{j=-1}^{n-1}\int dq_j\Pi_{j=0}^{n-1}\int(dp_j/2\pi\hbar)\cdot|q_{n-1}\rangle\langle q_{-1}|^n.\exp[-\int dt\langle\mathcal{L}(q_j;dq_j/dt)/i\hbar\rangle]$

$\equiv|f\rangle\langle i|\int Dq_j\int Dp_j.\exp[-\int dt\langle\mathcal{L}(q;dq/dt)/i\hbar\rangle]$..... **this is the orgin definition !!**

(4)Quantum Amplitude $=R_{fi}$ by Feymann Path Integral .

$\mathbf{S}(t;t_0)=|f\rangle\langle i|\int_{-\infty}^{\infty}Dq\int_{-\infty}^{\infty}Dp.\exp[-\int_{t0}^{t}dt\langle\mathcal{L}(q;dq/dt)/i\hbar\rangle]$.

$R_{fii}=\langle f|\mathbf{S}(t;t_0)|i\rangle$.

Operator part is $|f\rangle\langle i|$,the other are scalar term. This is not path-integral,but whole phase space one !.

[２]：Gauge Fixing by Path-Integral.

(1)$\mathcal{L}_{CF}\equiv -\frac{1}{4}(\partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu - f_b{}^a{}_c A^b{}_\mu A^c{}_\nu)^2 \equiv -\frac{1}{4} F^a{}_{\mu\nu} F^a{}_{\mu\nu}$.

$\mathcal{L}_{QF}\equiv \mathcal{L}_{CF}+\mathcal{L}_B=(ic)^{-1}B^a\partial_\mu A^a{}_\mu+\frac{1}{2}\alpha B^aB^a$.

(2)$0=(ic)^{-1}\partial_\mu A^a{}_\mu+\alpha{}^aB^a$.

The Euler Equation (2)must be gauge invariant by $\delta A^a{}_\mu$.

(3)$0=ic\partial_\mu A^a{}_\mu+\alpha B^a=ic\partial_\mu(A^a{}_\mu+\delta A^a{}_\mu)+\alpha B^a=ic\partial_\mu\delta A^a{}_\mu=ic\partial_\mu D_\mu\varepsilon^a=0$......

(4)$R_{CF}\equiv|f\rangle\langle i|\Pi_{a,\mu,x}\int DA^a{}_\mu\int D\Pi^a{}_\nu.\exp[-\int dx^4\langle\mathcal{L}_{CF}(A^a{}_\mu;\partial_\nu A^a{}_\mu)/i\hbar\rangle]$.

(5)**The Aim of Problem.**

At first,note that gauge transform never change observable physics.

In above,the integration $DA^a{}_\mu$ is to over-count due to gauge transform∞freedom,thereby **gauge fixing** by $\delta(ic\partial_\mu D_\mu\varepsilon^a)$ must be multiply the integral kernel to R_{CF} .However the compensation$=\Delta$ is simultaneously necessary toward being unity. $\int d\varepsilon\,\delta(k\varepsilon)=1/|k|$.

(5) $1=\Pi_{a,\mu,x}\int D\varepsilon^a\cdot\Delta\cdot\delta(ic\partial_\mu D_\mu\varepsilon^a)$.

(6)Review on **measure compensation=Jacobian** in integral variable transform.

http://tutorial.math.lamar.edu/Classes/CalcIII/ChangeOfVariables.aspx

$\Rightarrow y^b=f^b(t^1,t^2,...,t_N).\Leftrightarrow t^a=g^a(y^b)=f^{a-1}(y^b)$. <a, b=1, 2, 3, ..., N>

$\Pi_a dt^a=\Pi_b dy^b\cdot\det|\partial g^a/\partial y^b|=\Pi_b dy^b\cdot\det|\partial f^a/\partial y^b|^{-1}$.

$\Pi_a\int dt^a\delta[f^1(t^1,t^2,.),f^2(.),.]=\Pi_b\int dy^b\delta(y^1,y^2,.)\det|\partial f^a/\partial y^b|^{-1}=\det|\partial f^a/\partial y^b|^{-1}$.

$\Rightarrow 1=\Pi_a\int dt^a\delta(f^1(t^1,t^2,.,),f^2(.),.)\det|\partial f^a/\partial y^b|.\Rightarrow$ $\Delta=\det|\partial f^a/\partial y^b|$....(6)

(7)Deriving Δ.

(3)$0=ic\partial_\mu A^a{}_\mu+\alpha B^a=ic\partial_\mu(A^a{}_\mu+\delta A^a{}_\mu)+\alpha B^a=ic\partial_\mu\delta A^a{}_\mu=ic\partial_\mu D_\mu\varepsilon^a=0$.....

$ic\partial_\mu D_\mu\varepsilon^a\equiv f^a.\rightarrow\ \varepsilon^a=(ic\partial_\mu D_\mu)^{-1}f^a$.

$\rightarrow\ \Pi_{a,}D\varepsilon^a=\Pi_b Df^b\cdot\det|\partial\{(ic\partial_\mu D_\mu)|^{-1}f^b\}/\partial f^b\}|=\Pi_b Df^b\cdot\det|(ic\partial_\mu D_\mu)|^{-1}$.

$\rightarrow\ \Pi_{a,}D\varepsilon^a=\Pi_b Df^b\cdot\det|(ic\partial_\mu D_\mu)|^{-1}$. ....(7)'.

$\Rightarrow\ 1=\Pi_{a,x}\int D\varepsilon^a\cdot\delta(ic\partial_\mu D_\mu\varepsilon^a)\cdot\det|(ic\partial_\mu D_\mu)|.\ \Rightarrow\ \Delta=\det|ic\partial_\mu D_\mu|$....(7)

Above relation is to be changed as follows.This is very important.

☞:*Note we take technic<(7)'> in following doing integration on variables*$=\{\varepsilon^a;f^a=f^a(\varepsilon^a)\}$.

$1=\Pi_{a,x}\int Df^a\cdot\delta(-f^a)=\Pi_{a,x}\int Df^a\cdot\delta(ic\partial_\mu A^a{}_\mu-f^a)$

$=\Pi_{a,x}\int\Delta^{-1}\cdot Df^a\cdot\delta(ic\partial_\mu A^a{}_\mu-f^a)\cdot\Delta=\Pi_{a,x}\int D\varepsilon^a\cdot\delta(ic\partial_\mu A^a{}_\mu-f^a)\cdot\Delta$.

(8)Gauss Fresnel Integral Formula< $\int dx\cdot\exp(-ax^2/2)=\sqrt{(2\pi/a)}$>.

$*\int dx\cdot\exp(-iax^2/2)=\sqrt{(2\pi/ia)}$; $*\int dx\cdot\exp(ix^2/2a)=\sqrt{(2\pi ia)}$.

$\sqrt{(2\pi/ia)}=\int dB\cdot\exp(-ia(f/a+B)^2/2)=\int dB\cdot\exp[-i(f^2/2a+Bf+½aBB)]$

$=\int df\exp[-i(f^2/2a]\int dB\cdot\exp[-i(Bf+½aBB)]$.

$2\pi=\sqrt{(2\pi/ia)}\int df\exp[i(f^2/2a]=\int df\int dB\cdot\exp[-i(Bf+½aBB)]$.

$\to\quad 1=(2\pi\hbar)^{-1}\int df\int dB\cdot\exp[-i(Bf+½\alpha BB)/i\hbar]$. $\langle a=\alpha/\hbar;\ f'=f/\hbar\rangle$

(8) $1=\Pi_{a,x}\int Df^a\int dB^a\cdot\exp[\int dx^4(B^af^a+½aB^aB^a)/i\hbar]$.

By employing variable $f^a(\varepsilon^a)$,we are to do integral on the delta function.

(9)Gauss Integral Formula with Grassmann number= $\{\overline{C^a}, C^a\}$

https://en.wikipedia.org/wiki/Grassmann_integral

Grassmann number definition: $\overline{C^a}\cdot C^a+C^a\cdot\overline{C^a}=0$.

*This is **classical number-zation** of anti-commutable spinor ϕ.

$\phi*\phi+\phi\,\phi*=i\hbar\,\delta(\mathbf{x'}-\mathbf{x})$.

$\det\mathbf{A}=\Pi_{a,x}\int D\overline{C^a}\int DC^a\exp[\int dx^4\cdot\overline{C^a}(x)\,\mathbf{A}C^a(x)]$.

(9) $\Delta=\det\lvert ic\partial_\mu\mathbf{D}_\mu\rvert=\Pi_{a,x}\int D\overline{C^a}\int DC^a\exp[\int dx^4\cdot\chi\overline{C^a}(x)\cdot ic\partial_\mu\mathbf{D}_\mu C^a(x)/i\hbar$.

(10)**Total Quantized Lagragean of General Gauge Field.**

Ⅰ : $R_{CF}\equiv|f\rangle\langle i|\Pi_{a,\mu,\nu,x}\int DA^a_\mu\int D\Pi^a_\nu.\exp[\int dx^4\langle\mathcal{L}_{CF}(A^a_\mu;\partial_\nu A^a_\mu)/i\hbar\rangle]$.

Ⅱ : $\Delta=\det|ic\partial_\mu\mathbf{D}_\mu|=\Pi_{a,x}\int D\overline{C^a}\int DC^a\exp[\int dx^4\cdot\chi\overline{C^a}(x)\cdot ic\partial_\mu\mathbf{D}_\mu C^a(x)/i\hbar]$.

Ⅲ : $1=\Pi_{a,x}\int D\varepsilon^a\int dB^a\cdot\delta(ic\partial_\mu A^a_\mu-f^a)\exp[\int dx^4(B^af^a+½aB^aB^a)/i\hbar]\cdot\Delta$.

After all,multiplying (Ⅲ)×(Ⅰ)is to yield the total Lagrangean.

We do integration on the delta function by $\{D\varepsilon^a\}$.

$R_{QF}=\Pi_{a,\mu,\nu,x}\int D\varepsilon^a\cdot\int DA^a_\mu\int D\Pi^a_\nu\int DB^a\int D\overline{C^a}\int DC^a\cdot\exp[\int dx^4\langle\mathcal{L}_{CF}/i\hbar\rangle]\cdot$

$\delta(ic\partial_\mu A^a_\mu-f^a)\cdot\exp[\int dx^4(B^af^a+½aB^aB^a)/i\hbar]\cdot\exp[\int dx^4\cdot\chi\overline{C^a}(x)\cdot ic\partial_\mu\mathbf{D}_\mu C^a(x)/i\hbar]$

$=\Pi_{a,\mu,\nu,x}\int DA^a_\mu\int D\Pi^a_\nu\int DB^a\int D\overline{C^a}\int DC^a\cdot\exp[\int dx^4\langle\mathcal{L}_{CF}/i\hbar\rangle]$

$\cdot\exp[\int dx^4(ic\partial_\mu A^a_\mu B^a+½aB^aB^a)/i\hbar]\cdot\exp[\int dx^4\cdot\chi\overline{C^a}(x)\cdot ic\partial_\mu\mathbf{D}_\mu C^a(x)/i\hbar]$.

$=\Pi_{a,\mu,\nu,x}\int DA^a_\mu\int D\Pi^a_\nu\int DB^a\int D\overline{C^a}\int DC^a\cdot$

$\exp[\int dx^4\langle\mathcal{L}_{CF}+ic\partial_\mu A^a_\mu B^a+½aB^aB^a+\chi\overline{C^a}(x)\cdot ic\partial_\mu\mathbf{D}_\mu C^a(x)\rangle/i\hbar]$.

(10) $\mathcal{L}_{QF}=\mathcal{L}_{CF}+ic\partial_\mu A^a_\mu B^a+½aB^aB^a+\chi\overline{C^a}(x)\cdot ic\partial_\mu\mathbf{D}_\mu C^a(x)$. $\mathcal{L}_{CF}\equiv-¼F^a_{\mu\nu}F^a_{\mu\nu}\equiv-¼(\partial_\mu A^a_\nu-\partial_\nu A^a_\mu-f_b{}^a{}_cA^b_\mu A^c_\nu)^2$.