

The probability @ of outgoing infrared ray(cooling radiation from semi black body globe) is analyzed by a model of 1 dimensional GHGs atmosphere.

☞:The former page on RF has a fatal error model, which must be abandoned. So this version is correcting one. However also it is extremely simplified due to zero heat capacity of atmosphere comparing with that of oceans(depth nearly 1000m).

[0]:Radiative forcing(RF) is a deficit heat amount for input and output on earth.

(1)Heat Budget Equation on globe.

If solar ray input and output cooling radiation(≡CR) for globe were balanced, globe temperature  $T_G$  would not change. If the deficit(≡  $\delta F$ ) were,  $T_G$  would change.  $\delta F$  is defined as energy in(out)flow per unit area & time(watt/m<sup>2</sup>.sec). If  $\delta F > 0$ ,  $T_G$  would rise as Heat Budget EQN. Note  $T_G(t)$  is dominated by factors  $\{m(t), @ (t)\}$ . Our aim is to decrease  $T_G(t)$  by making  $\delta F < 0$ . **This is possible as the principle.**

$$\text{Heat Budget EQN: } \Delta Q_G(t) \equiv \text{input heat} - \text{output heat} \equiv 4\pi R_G^2 \delta F(t) = C_G(dT_G(t)/dt). \\ = \pi R_G^2 F_0(1-m(t)) - 4\pi R_G^2 @ (t) \sigma T_G(t)^4. \dots\dots\dots [0](1)$$

$I(0) \equiv \sigma T_G(t)^4$  is black-body radiation(cooling one≡CR) by globe without atmosphere(@ (t)=1). Then GHGs of atmosphere causes backward flow  $J(0)$  onto globe with rate  $b(t) \equiv 1 - @ (t) \equiv J(0)/I(0)$ .

(Global heat capa) ( $T_G$  rise/sec) =  $4\pi R_G^2$  (deficit heat) = input heat - output heat.

(2)geophysical constants:

\* $4\pi R_G^2 \equiv$  global surface area  $\doteq 4\pi (6.38 \times 10^6 \text{m})^2$ .

\* $C_G \equiv$  global heat capacity  $\doteq 1.29 \times 10^{24} \text{J/K}$ .

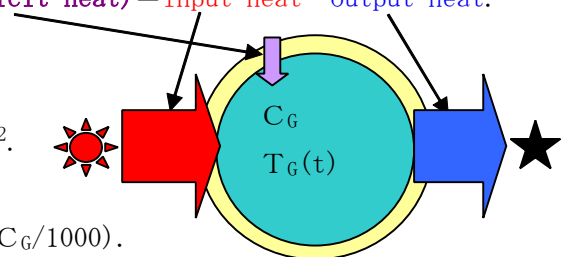
(all oceans(1000m depth) without atmosphere  $\doteq C_G/1000$ ).

\* $F_0 \equiv$  solar constant =  $1366 \text{W/m}^2 \cdot \text{s}$

\* $\sigma \equiv$  SB constant for blackbody radiation =  $5.67 \times 10^{-8} \text{W/m}^2 \text{K}^4$ .

\* $m(t) \equiv$  global albedo  $\doteq 0.3?$

\* $@ (t) \equiv$  CR outgoing probability  $\doteq 0.614?$



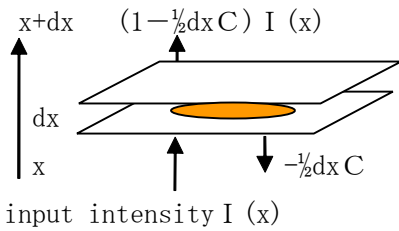
<< 0 dimensional global model >>

\*yellow zone is atmosphere of GHGs.

In the later, we show @ (t) is a function of GHG concentrations  $\{C_j\}$  which are operationable by us mankind. Albedo  $m(t)$  may be also same possibility.

**[1]:GHGs concentration {C} and the collision cross section for radiation beam.**

Concerned molecule gas global number "N" is defined in atmosphere volume  $V_A$  ( $C = N v_0 / V_A \equiv$  gas concentration with molecule volume= $v_0$ ). Assume differential segment layers of atmosphere at height  $(x, x+dx)$ . Then  $S dx$  is differential volume and  $C S dx$  is molecule volume with the area =  $C S$ . Hence **mean collision cross section** of molecules at  $(x, x+dx)$  is  $C$ /unit area (or **all the global surface**).



- (1) direct passing probability =  $1 - dx C$
- (2) trapped and upward-radiation =  $\frac{1}{2} dx C$
- (3) trapped and downward-radiation =  $-\frac{1}{2} dx C$
- (4) total passing probability =  $1 - \frac{1}{2} dx C$

Then note that collision beam on GHG are both upward  $I(x)$  and downward  $J(x)$ . Upper limit of integral domain in (5) need effective atmospheric height  $\equiv \langle H \rangle$ .

(5)  $C = V_c / V_A = N v_0 / V_A =$  particle number  $\times$  molecule volume / volume  $V_A$ .  $1 \text{ ppm} = 10^{-6} / \text{uv}$ .

(a) Ideal gas has same number of molecules in same volume:  $6.02 \times 10^{26} / 22.4 \text{ liter}$ .

\* molecule volume  $\equiv v_0 = 22.4 \times 10^{-3} \text{ m}^3 / 6.02 \times 10^{26} = 3.72 \times 10^{-29} \text{ m}^3 = (3.34 \times 10^{-10} \text{ m})^3$ .

\* Number of particles in  $1 \text{ ppm} / \text{m}^3 \equiv n_0 = 10^{-6} / 3.72 \times 10^{-29} \text{ m}^3 = 2.69 \times 10^{22} / \text{m}^3$ .

$\text{CO}_2$  (385 ppm) weight /  $\text{m}^3 = (44 \text{ g} / 6.02 \times 10^{26}) \times 385 \times 2.69 \times 10^{22} / \text{m}^3$ .

(b) total air weight  $\equiv M_T = P (g \times 1 \text{ atm}) \times 4 \pi R_E^2 / g = 1033 \text{ kg} \times 4 \pi (6.38 \times 10^6 \text{ m})^2 = 5.28 \times 10^{17} \text{ kg}$

(6) effective atmospheric height  $\equiv \langle H \rangle: V_A \equiv \langle H \rangle \times 4 \pi R_G^2$ .

Half the atmosphere is within 6 km of the surface, 90% is within 20 km.

<http://www.exo.net/~pauld/workshops/Greenhouse%20Effect/greenhouse.html>

**[2]:Primitive model building and the analysis:**

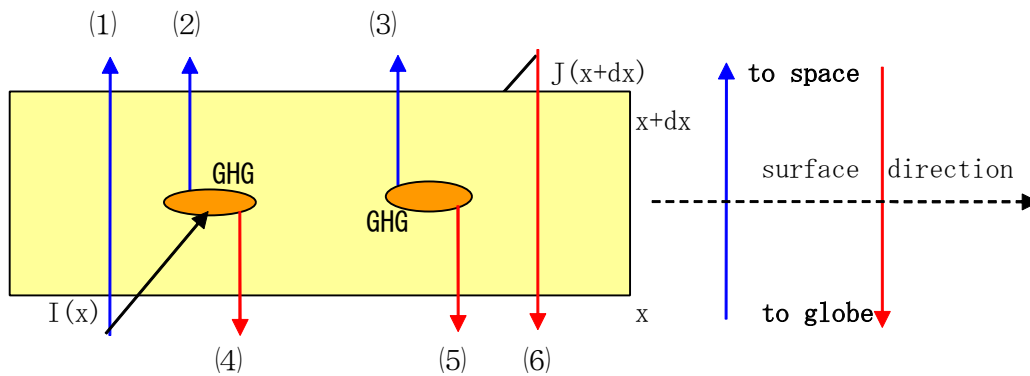
The fundamental bad function of GHG is heat trapping without escaping to space. We assume an atmosphere without heat capacity, that is, nothing heat reserving. We assume also steady state flow of outgoing and incoming infrared ray radiation. This assumption is due to **the zero dimensional global climate model**, where substantila heat capacity is that of oceans  $C_G$ , while atmosphere is  $C_G / 1000$ . As the first step, we assume CR beam penetrating process without spectrum density. Then  $\{ \alpha dx \}$  is differential heat trapping probability between  $(x, x+dx)$ . Trapped radiation has equal probability =  $\frac{1}{2} \alpha dx$  for outgoing and incoming direction.

① **Outgoing flow** intensity  $I(x)$  from  $x$  to  $(x+dx)$ :

- (1) direct passing radiation =  $(1-\alpha dx)I(x)$ .
- (2) trapped and reradiation to space =  $+\frac{1}{2}\alpha dx I(x)$ .
- (3) trapped and reradiation to globe =  $-\frac{1}{2}\alpha dx I(x)$ .

② **Backward flow** intensity  $-J(x)$  from  $(x+dx)$  to  $x$ :

- (4) direct passing radiation =  $-(1-\alpha dx)J(x+dx)$ .
- (5) trapped and reradiation to space =  $+\frac{1}{2}\alpha dx J(x+dx)$ .
- (6) trapped and reradiation to globe =  $-\frac{1}{2}\alpha dx J(x+dx)$ .



③ **budget in space side  $(x+dx)$ .**

$$K(x+dx) \equiv I(x+dx) - J(x+dx) = (1-\frac{1}{2}\alpha dx)I(x) + \frac{1}{2}\alpha dx J(x+dx) - J(x+dx)$$

$$I(x+dx) = (1-\frac{1}{2}\alpha dx)I(x) + \frac{1}{2}\alpha dx J(x+dx).$$

$$I(x) + dx(dI/dx) = (1-\frac{1}{2}\alpha dx)I(x) + \frac{1}{2}\alpha dx \langle J(x) + dx(dJ/dx) \rangle.$$

$$dx(dI/dx) = -\frac{1}{2}\alpha dx I(x) + \frac{1}{2}\alpha dx J(x). \rightarrow dI/dx = \frac{1}{2}\alpha \langle J(x) - I(x) \rangle.$$

④ **budget in globe side  $(x)$ .**

$$K(x) \equiv I(x) - J(x) = I(x) - (1-\frac{1}{2}\alpha dx)J(x+dx) + \frac{1}{2}\alpha dx I(x)$$

$$J(x) = (1-\frac{1}{2}\alpha dx)J(x+dx) + \frac{1}{2}\alpha dx I(x) = (1-\frac{1}{2}\alpha dx) \langle J(x) + dx(dJ/dx) \rangle + \frac{1}{2}\alpha dx I(x)$$

$$= (1-\frac{1}{2}\alpha dx)J(x) + (1-\frac{1}{2}\alpha dx)dx(dJ/dx) + \frac{1}{2}\alpha dx I(x).$$

$$0 = -\frac{1}{2}\alpha dx J(x) + dx(dJ/dx) + \frac{1}{2}\alpha dx I(x). \rightarrow dJ/dx = \frac{1}{2}\alpha \langle J(x) - I(x) \rangle.$$

$$\textcircled{5} d \langle J(x) - I(x) \rangle dx = 0. \rightarrow J(x) - I(x) = \text{constant} \equiv bI(0) - I(0).$$

$$J(0) \equiv bI(0) : \text{GHG pseudo reflecting probability at surface. } \langle *b \equiv J(0)/I(0) \rangle.$$

$$J(x) = I(x) + I(0) \langle b-1 \rangle \equiv I(x) - @I(0). \langle *@ \equiv 1-b : \text{passing probability to space} \rangle.$$

$$dI/dx + \frac{1}{2}\alpha I(x) = \frac{1}{2}\alpha \langle I(x) - @I(0) \rangle. dI/dx + \frac{1}{2}\alpha I(x) = \frac{1}{2}\alpha I(x) - \frac{1}{2}\alpha @I(0).$$

$$dI/dx = -\frac{1}{2}\alpha @I(0). \quad I(x) = -\frac{1}{2}\alpha @I(0)x + C.$$

$$I(x) = -\frac{1}{2}\alpha @I(0)x + I(0) = I(0) [1 - \frac{1}{2}\alpha @x].$$

$$0 = J(H) = I(x=H) - @I(0) = I(0) [1 - \frac{1}{2}\alpha @H] - @I(0), \text{ where "H" is effective height.}$$

$$@ = [1 - \frac{1}{2}\alpha @H]. \quad @(1 + \frac{1}{2}\alpha H) = 1. \quad 1 + \frac{1}{2}\alpha H = 1/@. \quad \frac{1}{2}\alpha H = (1/@ - 1). \rightarrow \alpha = 2(1/@ - 1)/H.$$

$$I(x) = I(0) [1 - (1 - \alpha)(x/H)]. \quad J(x) = I(x) - \alpha I(0) = I(0) [1 - (1 - \alpha)(x/H)] - \alpha I(0)$$

$$= I(0) \{ [1 - (1 - \alpha)(x/H)] - \alpha \} = I(0) \{ [1 - \alpha - (1 - \alpha)(x/H)] \} = I(0) (1 - \alpha) [1 - (x/H)].$$

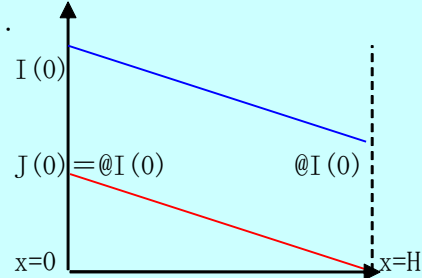
⑥ **Outgoing and backward intensity with  $\{\alpha, \alpha\}$ .**

(1)  $I(x) = I(0) [1 - (1 - \alpha)(x/H)].$

(2)  $J(x) = I(0) (1 - \alpha [1 - (x/H)]).$

(3)  $\alpha = 2(1/\alpha - 1)/H.$

(4)  $\alpha = 1/(1 + \alpha H/2).$



**[3]: GHGs concentration  $\{C_j\}$  with molecule spectrum window function  $\{S_j(\nu)\}$ .**

$\alpha$  is determined by cross section  $\alpha$ /unit area.  $\alpha$  is function of GHG concentration  $\{C_j | j=1, 2, \dots, N\}$  with the molecule spectrum window function  $\{S_j(\nu) | j=1, 2, \dots, N\}$ .  $S_j(\nu)$  is spectral **absorbtion** function of a molecule of kind  $\{j\}$  spectroscopy.

(1)  $d_\nu I(x; \nu) C_j S_j(\nu) \equiv$  differential flux  $d_\nu I(\nu)$  trapping probability on molecule  $\{j\}$

$$I(x) \rightarrow d_\nu I(x; \nu); J(x) \rightarrow d_\nu J(x; \nu);$$

$$\alpha(\nu) \equiv \sum_{j=1}^N C_j S_j(\nu), \quad \langle\langle \text{assumption on linear sum of concentration} \rangle\rangle$$

$$\alpha(\nu) \equiv 1/(1 + \alpha(\nu)H/2).$$

(2)  $dI(x, \nu)/dx = \frac{1}{2} \alpha(\nu) \langle J(x; \nu) - I(x; \nu) \rangle;$

$$dJ(x, \nu)/dx = \frac{1}{2} \alpha(\nu) \langle J(x; \nu) - I(x; \nu) \rangle.$$

(3)  $I(x; \nu) = I(0; \nu) [1 - \langle 1 - \alpha(\nu)(x/H) \rangle].$

$$I(x) \equiv \int_0^\infty d_\nu I(x; \nu) = \int_0^\infty d_\nu I(0; \nu) [1 - \langle 1 - \alpha(\nu)(x/H) \rangle].$$

$$I(0) \equiv \int_0^\infty d_\nu I(0; \nu) = \int_0^\infty d_\nu I(0; \nu).$$

Now we take into account of time dependency of  $\{I(t), C_j(t)\}$ .

$$I(t; H) \equiv \int_0^\infty d_\nu I(t; H; \nu) = \int_0^\infty d_\nu I(t; 0; \nu) \alpha(t; \nu) \equiv \alpha(t) \int_0^\infty d_\nu I(t; 0; \nu)$$

$$= \int_0^\infty d_\nu I(t; 0; \nu) / [1 + (H/2) \sum_{j=1}^N C_j(t) S_j(\nu)].$$

(4): **CR passing probability**  $\langle \text{macroscopic pass} \equiv \alpha(t) \rangle.$

$$\alpha(t) \equiv I(t; H) / I(t; 0)$$

$$= \int_0^\infty d_\nu I(t; 0; \nu) / [1 + (H/2) \sum_{j=1}^N C_j(t) S_j(\nu)] / \int_0^\infty d_\nu I(t; 0; \nu).$$

The main factor of GHG effect is represented by  $\alpha(t)$ , which is time function of time dependent GHG concentration  $\{C_j(t)\}$  with outgoing flux  $I(x=0)$ . The global warming depends on both main factor  $\{m(t), \alpha(t)\}$ .  $\alpha(t)$  is evidently a decreasing function of heat trapping gass concentration  $\{C_j(t)\}$  increasing, which in turn causes  $T_G(t)$  rise in [0](1)  $\langle \text{Equation of Global Temperature} \equiv \text{EGT} \rangle.$