## Uncertainty Causes in Climate Dynamics. 2016/10/4,12.14,15,17,19

Climate variables such as temperature of averaged value are causalstic, so predictable by the theory,while **an actual realization** fluctuates around the averaged value with the definite probability which is also predictable by theory. At least,such theory must exist.

(1)The troublesome observable big variation(fluctuation)of climate variables at local spots=x at time=t is almost due to climate fluid behavior(Fluid Dynamics)in atmosphere and ocean. An evidence is showed in emerging **visible macroscopic TUBULENCE** by the mechanics.
(2)A regional or global climate variables are considered as those additive sum effect and by the consequence, also those variables could be fluctuate<CENTRAL LIMIT THEOREM>
This report is to show you (1)(2) in order to make assured climate actions with prediction.
☞:Also **predictable periodic insolation variation** is added the fluctuation.

| |
|---|
| Climate Fluctuations(temperature,wind intensity,insolation,rain fall,..) |
| almost **Central Limit Theorem** for additive Random Variables in larger dimensional space |
| **Fluid Turbulence** the origin of probability in local atmosphere,and oceans. |

☞ : readers should be familiar with physics and math the fundamental.(2) is rather advanced course,so you should learn **the decisive conclusion** only.

## Part Ⅰ : Statistical Theory the Introduction and Climate Examples.

### [１]:Statistical Climate Prediction<the fundamental>. 2016/9/23

⑴**Definition on Statistical Ensemble of many Observable Samples.**

Each dice top surface value is **a realization as a sample** in the **Statistical Ensemble** ≡ {1,2,3,4,5,6} ,which is called also **random variables(RV)** ≡{X }and **each realization (sample )**has proper **probability(distribution)**such as P(X=1)=1/6,........., P(X=6)=1/6

*__average value__ **<X>** ≡all sum of (probability×random variable)＝21/6＝3.5.

*__fluctuation__ δX≡ (a realization=X)－(average**<X>**)＝2-<3.5>＝*-1.5* ; *6*-<3.5>＝*+2.5*

☞:the most remarkable feature of random variables{ δX}tends to become **zero** by the summation in actual N times observation,that is averaging { $\delta X_k$,k=1,2,3,...,N} is zero. $N\rightarrow\infty\{\Sigma_{k=1,}{}^{N}\delta X_k\}/N=0$.

*A random is unpredictable,*while this fact is very useful in prediction stage.Because the random realization distribute symmetric between +side and -side.It never be biased one side.Those are to zero cross alternately,however the time is random.It is so to say a **NOISE wave**.

☞:**Once again difference between a realization and statistical ensemble !!.**

Russian roulette is a deadly trial,while repeatable dice throwing is trial**s** in an ensemble.

A realization is entirely unique & only,while **Statistical Theory Prediction** assumes **virtual reality** of repeatable trials in **the same defined ensemble**.

⑵**a concrete example) temperature T at positon=x,time=t<Stochastic Process>.**

An averaged temperature≡〈T(**x**;t))> in time is ruled by **something causalstic mechanism(THEORY)**,while **a realization**≡T(**x**;t) in past or coming future is randomly fluctuated by δT(**x**;t) from the average value 〈T(**x**;t)>with something **probability.**

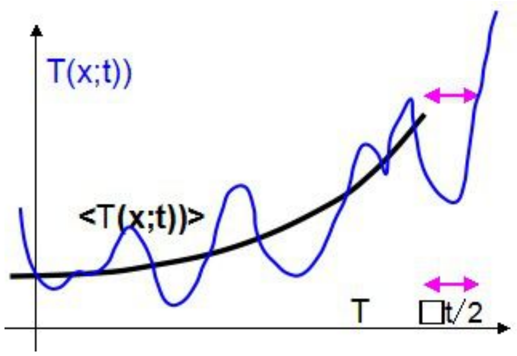
**∗an actual realization**≡T(**x**;t)≡〈T(**x**;t)〉＋ δT(**x**;t)。

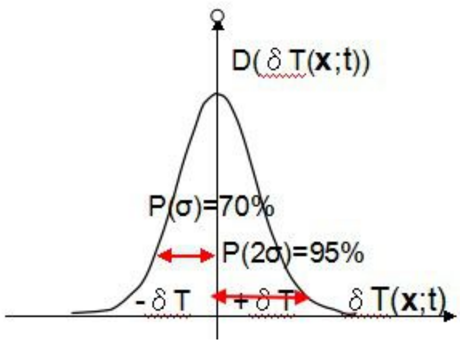
**∗averaged value by a time interval** (□t)≡〈T(**x**;t)〉≡$\square t^{-1}\int_{t-\square t/2}^{t+\square t/2} dtT(\mathbf{x};t)$.

***∗Fluctuation probability is same as Prediction Probability !!***

Probability of a≦ δT≦b= $\int_a^b d(\delta T).D(\delta T)=\int_a^b du.D(u)$..

D( δT) is called probability density function(PDF),and d( δT) is infinitesimal width.



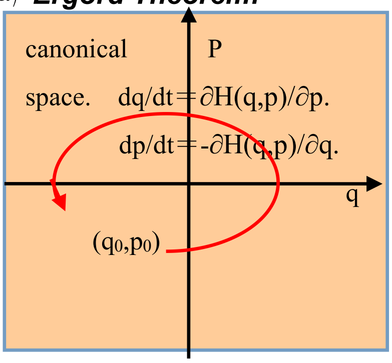



### (3)Time Interval Average＝Statistical Ensemble Average ??!!.

This is very serious big theme in actual Climate Observation.

Because it is **averaged trend**(in noisy process)that could derive casalstic mechanism (predictability).So called ***Ergord Theorem*** in **Equilibrium Statistical Mechanics** affirms it,while climate is non-Equilibrium !,But,**in a short time interval averaging** ,climate may be **near Equilibrium State ?!.** In fact,there is no other method,but it ,and many affirm it.

(a) ***Ergord Theorem:***




System is equilibrium state with massive particles with (q,p).

(q,p)is dynamic variable of position and momentum of each particles of which trace is ruled by **dynamic equation**.

**Statistical Ensemble** is defined as $\rho$ (q,p) the PDF.

$<A(t_0)>= \int dq_0,dp_0,\ \rho(q_0,,p_0,)A(q_0,,p_0,),$

While the theorem proves that $\rho(q_0,,p_0,)=\rho(q(t),,p(t),)$

$<A(t_0)>=<A(t)>=(1/T)\int_T dtA(q(t),p(t)),$

☞: $S(t)=k_B\int dqdp.\ln(1\ \rho(q(t),p(t)))=S(t_0)$。

This is **equilibrium state** definition itself by max entropy in thermodynamics.

Above proof is due to **classical dynamics** in canonical formulation,but in quantum one(QD),however the theorem could be told valid also in QD.

(b)Climate system is not **equilibrium state,**but slowly time dependent,however it could be considered **equilibrium**,in short time interval(5 years,or 11ones).In fact,there is no other averaging method,but it ,thereby many can not help,but affirm it. That is,it is approximation method,so we must estimate the statistical fluctuation PDF.

.

(c)If trace=T(t)was ruled both by **causalstic evolution EQN** with **random noi**se=ΔT(t), what happen ?.Our aim is estimation on time trend estimation on dΔT(t)/dt.

This serious problem shall be mentioned in following theme in this site before long.

However this could not be complete,but a proposing toward the big problem solution.

***Global Surface Temperature Fluctuation Analysis*.**

The conclusion by observing actual global temperature trend(1880~2016)is stationary(?) Gaussian Fluctuation(PDF) with something adding causalstic periodic effects.

Gaussian PDF is due to central limit theorem in **fluid turbulence** in climate environment.

Note clouds & humidity in turbulence field becomes dominant role,research of which proof is task of the experts.

［2］: **Microscopic Cause of Fluctuation**<Random Walk the primitive,but a supreme model>
Our concern is fluctuation,one of which simplest conclusion is here(Random Walk).

(a)**Random Walk the definition and the model:**

dx is infinitesimal width where N=dx $\rho(x;t)$ random walk particles are gathered.

Those are to flow(walk) to both left and right with same probability 1/2 in certain rate.

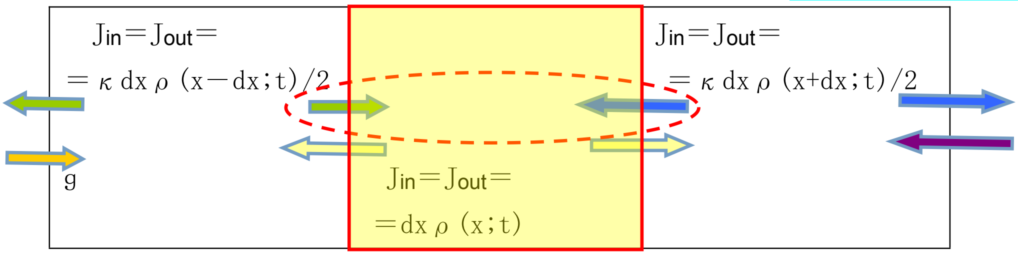
Note this 1/2 is the suprem uncertainty for the dynamism,the least information !!!.

This is the definition of 1 dimensional **random walk** and below is the model schme.

This is also analogy of **diffusion process** in statistical mechanics.

Then in and out flow at x is proportional to particle number N/2＝dx $\rho(x;t)/2$,

That is,the flow is $\kappa$ dx $\rho(x;t)/2$,where $\kappa$ is constant.<＊ $\rho$ (1- $\kappa$ )is not to move!>




Now we derive substantial flow at x by total flow summation at x box,thus

＊ $dxJ(x;t)=-\{\kappa dx\rho(x+dx;t)/2-\kappa dx\rho(x-dx;t)/2\}=-\kappa dx\partial\rho(x;t)/\partial x.$

*Flow is proportional to density gradient.*

$\partial\rho(\mathbf{x};t)/\partial t=-\mathrm{div}\mathbf{J}(\mathbf{x};t).$ 〈Conservation law of flow and density〉.

$\mathbf{J}(\mathbf{x};t)=-\kappa\,\mathrm{grad}\,\rho(\mathbf{x};t).$ 〈flow due to density gradient〉

$\partial\rho(\mathbf{x};t)/\partial t=\kappa\,\mathrm{divgrad}\,\rho(\mathbf{x};t)=\kappa\nabla^2\rho(\mathbf{x};t).$ 〈**Diffusion Equation**〉

＊ $D\mathbf{V}(\mathbf{x};t)/Dt=\kappa\nabla^2\mathbf{V}(\mathbf{x};t)+\mathbf{f}.$ 〈**Fluid Dynamics Equation**〉

☞:Note the similarity of being diffusion term !!!.which is to cause chaos in the equation.

(b)**Probability Distribution Function in Random Walk or Diffusion .**

$\rho(\mathbf{x};t)=\rho_0(4\pi\kappa t)^{-1/2}.\exp[-x^2/4\kappa t].$ **Normal Distribution the solution.**

$\sigma^2\equiv\langle(x-\langle x\rangle)^2\rangle=2\kappa t.$ **Deviation Time Dependency.**

☞: $\kappa$ is dominant ruler in $\sigma^2$.

(c)**The Kernel Meaning of Nothing Mechanism,but Random at all.**

Random walk is decisive reality of **nothing causalstic mechanism** in nature.

This could be supreme chaos in causalstic prediction.Because not full random means (full random part)＋(causalsticl part). After all we could not evade being of full random.

**Atmospheric** and **Ocean flows** in climate never evade this reality due to the massive random molecular particle nature. The being is to interfere complete causalstic prediction.

## [3]：MACROSCOPIC Cause of Fluctuation in RV multidimensional space.
## <Central Limit Theorem by variables(=RVs) in larger dimensional space>

Central Limit Theorem states that the sum of N random variables with same PDF will approach normal distribution as N approaches infinity. Random walk-itself is a typical due to additive of random walk length variables.Even N is small,those approach near Gaussian. Climate variables become such random variables,which is to mentioned in below.

Our most concern is that many of **climate fluctuations** may be due to those nature.

### (1)CENTRAL LIMIT THEOREM:

http://www.true.net/intro-stat.pdf

$\{f_k(X_k) | k=1,2,..,n\}$is arbitrary distribution function with definite mean and deviation value $\{<X_k>;<\sigma_k>\}$. Then additive random variable is defined.：Then $n \to \infty$ is Normal Distribution.

$f(x)=N[n\to\infty, X\equiv\sum_{k=1}^{n} X_k/n\,;<\sigma^2>\equiv\sum_{k=1}^{n}<\sigma_k^2>/n\,]$.

☞ $:<\sigma^2>n\equiv<\sigma^2>,\to\sigma^2=\sum_{k=1}^{n}\sigma_k^2$. this is **space length** in n dimensional random variable space.

(2)Note integral calculation is also additive in $n\to\infty.X_k\to 0$

That is infinitesimal random variables sum could be RVs in **infinitive dimension**.

We could derive following discrete to integral formulation.

$X\equiv(b-a)^{-1}[\sum_{k=1}^{n}x_k(b-a)/n].,\to n\to\infty, X=(b-a)^{-1}\sum_{k=1}^{n}x_k(b-a)/n=(b-a)^{-1}\int_a^b du X(u)$.

$*n\to\infty,(b-a)/n\equiv du;X(u\equiv a+k(b-a)/n)=x_k.<k=n(u-a)/(b-a)>$.

**Additive Random Variables** in ∞ dimension random variables space.

$X(t)=(b-a)^{-1}\int_a^b du X(u;t))$.

This is so to say a stochastic integral form,and is pragmatical in actual climate case.

<see Part I [4]：**Example-1) Sea Surface Temperature**>.

(3)**Almost Central Theorem:**This is also very simple and useful theorem in actual case.

*Finite dimensional random variables sum of Gaussian PDF is also Gaussian.*

$Z=X+Y$、$X\equiv\sum_{k=1}^{n}X_k/n, Y\equiv\sum_{k=1}^{m}Y_k/m$.

$G(x)=1/\sqrt{(2\pi)}\sqrt{\sigma_X^2}\cdot\exp[-(x-m_X)/2\sigma_X^2].;G(Y)=1/\sqrt{(2\pi)}\sqrt{\sigma_Y^2}\cdot\exp[-(x-m_Y)/2\sigma_Y^2]$.

$H(Z=X+Y)=\int_0^{\infty}dxG(x)G(z-x)=1/\sqrt{(2\pi)}\sqrt{(\sigma_X^2+\sigma_Y^2)}\cdot\exp[-(z-(m_X+m_Y))/2(\sigma_X^2+\sigma_Y^2)]$.

Note If each X and Y are variables in central limit theorem one,

**each original random variables** of X and Y can have each different PDF !!!,

If elements random variable are infinitesimal,their infinitive sum becomes Gaussian.

citation:K.Udagawa,Introduction to Applied Probability Theory,Ω Publishing Co,1964.

## [4]: Actual Climate Case Study.

(1)**Example-1) Sea Surface Temperature**=T(**x**,t) at some region=**x**,at time=t.

dQ(**x**,t)/dt＝c(**x**)<dT(**x**,t)/dt>＝－div(**V**(**x**,t)Q(**x**,t))..................Heat conservation local law.

∰dVc(**x**)<dT(**x**,t)/dt>＝－∰dVdiv(**V**(**x**,t)Q(**x**,t))＝－∯dS. **V**(**x**,t)Q(**x**,t)＋$J_B$(**x**,t)..

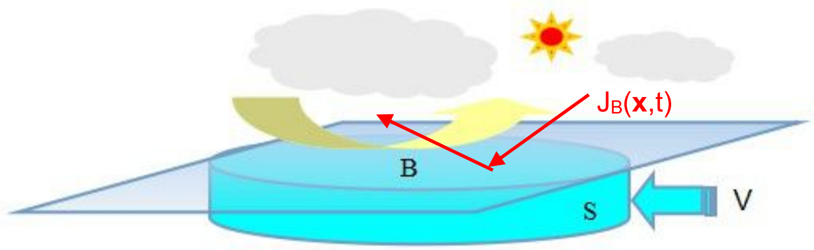
(C(**x**)<dT(**x**,t)/dt>＝∯dS<－**V**(**x**,t)Q(**x**,t)＋$J_B$(**x**,t)>................Heat conservation global law.

<T(**x**,t)>＝<T(**x**,$t_0$)>＋(1/C(**x**)){∫$dt_{t0}{}^t$∯dS<－**V**(**x**,t)Q(**x**,t)＋$J_B$(**x**,t)>}

＊C(**x**)≡∰dVc(**x**)...............Heat Capacity of the region(x).

**V**(**x**,t)Q(**x**,t) ≡**J**$_Q$(**x**,t) is random variables of heat flow at surface=S(x),which is due to randomness of flow **V**(**x**,t), $J_B$(**x**,t) is another complex heat flows at boundary=B.

<T(**x**,t)> is random those sum as time and space integral.




☞ : Note mechanism of $J_B$(**x**,t) and －**V**(**x**,t)Q(**x**,t) may be not the same,and the PDF may be different. Then could central limit theorem be valid ?<see [3]: (3)>,.
Those each elements could have different PDF,however final values is both sum.
∯dS<$J_B$(**x**,t) is also additive some which must be Gaussian.After all,all are Gaussian.

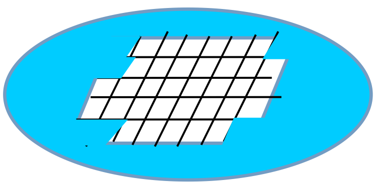
### (2)Fluid Path is to Fluctuate by Additive Sum of Local Random Forces !.

**Macroscopic View on Fluid Dynamics and the Fluctuation Cause.**
Following is Fluid Equation as global surface=S<**APPENDIX_5** : >.

∰dV・D(ρv)/Dt＝－μ∯dS×curl**V**＋[μ∯dS・div**V**＋∯dS・**P**]＋∰dV・ρ**K**.

Turbulence(PartⅡ) may be local in climate fluid field,however above surface integral is massive sum of driving forces in fluid path space and time(the large and long).Thereby the path could be fluctuated by CENTRAL LIMIT THEOREM. Above is so to say a stochastic integral.

(3)**Example-2) Sea Ice Extent** at time=t in Arctic.



Arctic sea ice extent is decisive in our destiny,
which is **random sum of each bit area of cells**.
The large fluctuation is remarkable.

⑷the actual case study<global temperature trend and wild climates>.

http://www.columbia.edu/~jeh1/mailings/2016/20160926_BetterGraph.pdf

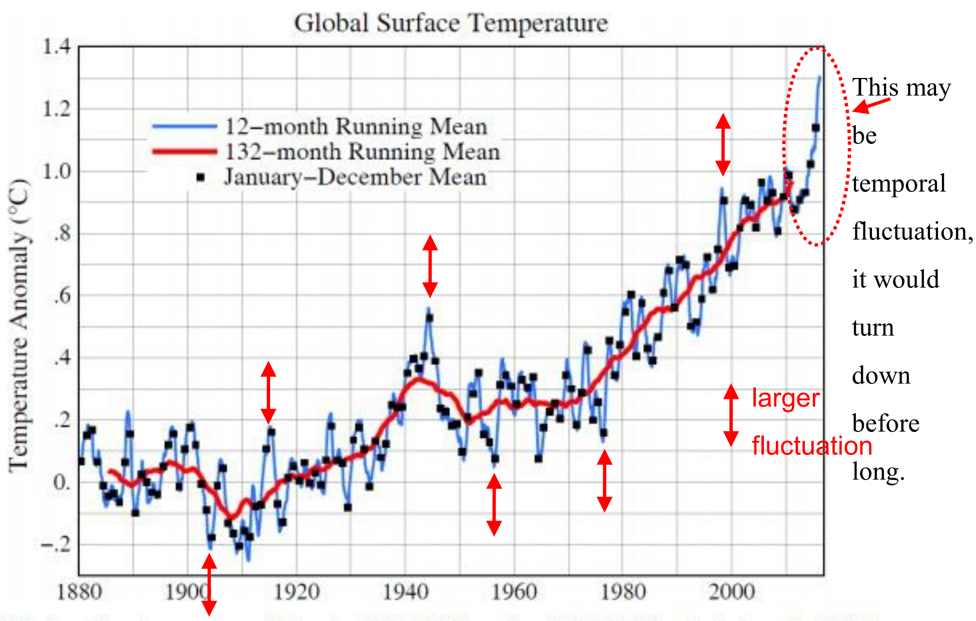



Fig. 1. Global surface temperature relative to 1880-1920 based on GISTEMP analysis (mostly NOAA data sources, as described by Hansen, J., R. Ruedy, M. Sato, and K. Lo, 2010: Global surface temperature change. *Rev. Geophys.*, **48**, RG4004. We suggest in an upcoming paper that the temperature in 1940-45 is exaggerated because of data inhomogeneity in WW II. Linear-fit to temperature since 1970 yields present temperature of 1.06°C, which is perhaps our best estimate of warming since the preindustrial period.

＊recent liner trend(by Suzuki)=<1.15(2015)－0.75(2000)>℃/15y＝0.027℃/y.

<1.10(2015)－0.55(1990)>℃/25y＝0.022℃/y.

Certainly this liner trend estimates about **0.3℃rise in decade**,which is dangerous.

http://www.777true.net/0.1C-Temperature-Rise-could-cause-Climate-Wild.pdf

Thereby,urgent massive CO2 extraction is decisive to **TURN** the bad trend. !!!

It is certain that without consideration on the absolute value temperature,below is nonsense.

If year fluctuation goes beyond 0.3℃(???2σ→5%),the energy amount is equivalent to 1400 Hurricanes.Even the 1%? partition rate,it becomes 15 Hurricanes **increase**.

＊0.1℃Global Temperature Rise≒$8.6\times10^{22}$Joule heat energy increase

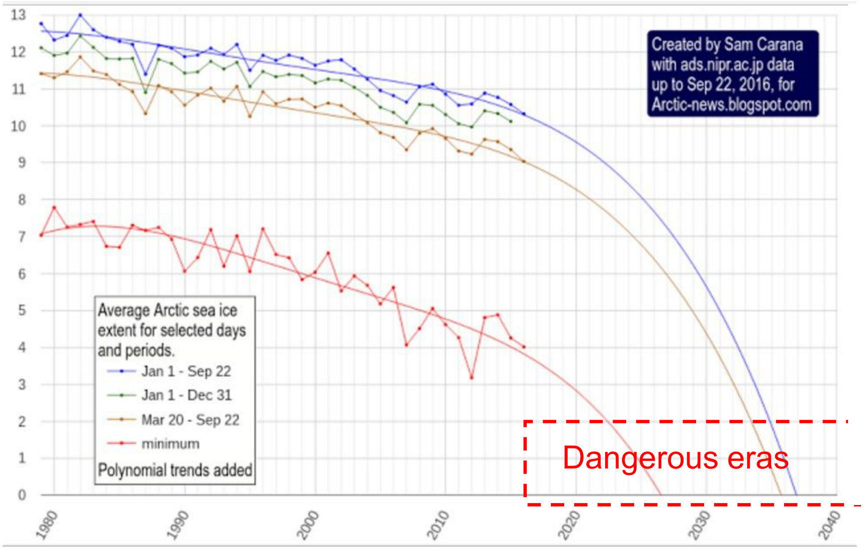
＊How much energy for making a hurricane ?≒$1.8\times10^{20}$Joule.

http://www.777true.net/0.1C-Temperature-Rise-could-cause-Climate-Wild.pdf

☞:annual global heat increase must be **partitioned** to many heat reservor,such as ocean (depth=600~700m)by 99%,atmosphere and land.At the initial time,those could not be **thermal-equilibrium in global,**so it must need **stirring** of atmospheric and ocean flows in **local** toward the **global equilibrium**.Note also **the partition rate**-itself(1%?) may have big fluctuation.

⑸the actual case study<Arctic Sea Ice Extent>.

http://arctic-news.blogspot.jp/p/arctic-sea-ice.html




Fluctuation at max extent is less, while that of minimum is larger.Becoming thin in hidden **ice thickness** is serious(average thickness is told about only 2m !).

## Global Heat Budget in Arctic and Sea Ice Extent Annual Decline(＝$dS_I/dt$).

This is annual account for Global Arctic.However author can not assure the exactness.

http://www.777true.net/Rapid-Temeperature-Rise-in-Arctic-a-simple-verification.pdf

| | Input | Output | Annual reseve |
|---|---|---|---|
| Coolig Radiation output(a) | | $115W/m^2$ | **These values=**Arctic total/$S_AY$, |
| Insolation input(b) | $30W/m^2$ | | Y=365x24x3600sec. |
| Ocean heat input(c) | $6W/m^2$ | | $1m^2$ space and 1sec time scaling. |
| Atmospheric heat input(d) | $84W/m^2$ | | |
| total input－ total output | $120W/m^2$ | $115W/m^2$ | $5W/m^2$ |
| Ocean warming(e) | $C_S(dT/dt)=4.5W/m^2$ (dT/dt＝0.031℃/y) | | |
| Land & atmospher warm(f) | | | |
| Sea ice decline＝$dS_I/dt$ | $-L\rho_I Q_I(dS_I/dt)=0.5W/m^2$ | | |

*Arctic Ocean(=AO) Area=$S_A$=$14.7x10^{12}m^2$. ☞:$dS_I/dt$ is annual area decreasing rate.

*AO Dynamic Heat Capacity/$m^2$=$C_S$=$79W/m^2K$, density=$1020kg/m^3$,specific heat=$4.02x10^3$J/kg,

*Ice latent Heat＝$Q_I$＝334.7KJ/Kg, $\rho_I$＝ice mass density=$917Kg/m^3$,

*ice area $S_I$＝{ice Volume/mean ice thickness(L=2m)}/$S_A$,

☞:Sea ice decline is function of random variables of ⑶. Other serious influence are **random Arctic climate(wind,storm,..)**,and **random ocean current**,Generating ice cracking accelerates ice diminishing,which is also **random phenomena** around average value.However **long years trend estimation** may be causalsticl<however ,author don't know well>.

**Part Ⅱ：Origin of Climate Fluctuation＝Fluid Turbulence Erasing the Trace.**

**<Fluid Equation as Stochastic One>.** 2016/9/25,10/12

[1]：**Viscosity=**$\mu$ in Fluid Dynamics Equation is random in turbulence.

Our concern is what is the cause of macroscopic climate fluctuation in the prediction.Also author's conclusion is fluid dynamics with chaotic turbulence term of $\mu\nabla^2\mathbf{V}$. Certainly $\mu$ in turbulence is to realize **visible macroscopic chaotic behavior** of fluid. The evidence is massive heat generation in fluid. $\mu$ is not pure constant, but rather random variable.

(1)*k-ε MODEl*(an empirical model theory).

http://www.cradle.co.jp/tec/column04/

$\mu_t = C_\mu . k^2/\varepsilon$ . <Turbulence energy≡k ; Turbulence vanishing rate≡ $\varepsilon$ ;$C_\mu$ ≡model constant>.

(a)**Turbulence Energy(Heat Loss)=Force×Velocity of Ship.** <by Suzuki>

$-L \equiv \oiint_D dV\,\mathbf{V}\cdot\mu\nabla^2\mathbf{V} = \mu\oiint_D dV[\mathrm{div}(\mathbf{V}\,\mathrm{div}\mathbf{V}-\mathbf{V}\times\mathrm{curl}\mathbf{V})-(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2]$

$= -\mu\oiint_D dV[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2]$.

Ship Driving Force×the Velocity＝L≡fluid energy consumption by unit time in volume D＝

＝(vanishing by surface integral)

＋(**thermal loss by jet blow**)＋(**thermal loss by fluid stirring**).

http://www.777true.net/Information-Loss-Process-in-NS-Equation_The-Cause-of-Chaos.pdf

A jet blow is frontal(backward)collision of fluid mass,while fluid stirring is side rubbing collision of fluid mass.Those collisions can't be causalstic,but probabilistic due to Quantum Theory

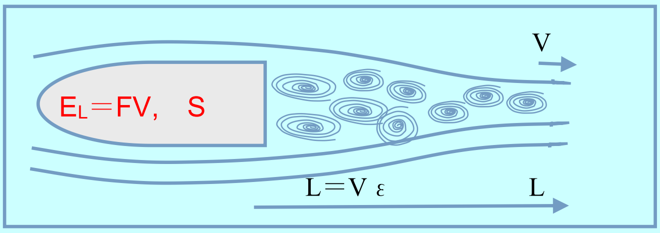
$\rightarrow \mu_t = L/\oiint_D dV[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2] \equiv L\langle(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2\rangle^{-1}/D \equiv C_\mu . k^2/\varepsilon$ .

$*\ (1/D)\oiint_D dV[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2] \equiv \langle(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2\rangle$=average watt density.

.

$k^2 \equiv \langle(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2\rangle^{-1}$. ....... 1/**turbulence energy density**

(b)$C_\mu \equiv L$. ....... **model constant.**




*A model may be a test run flow toward industrial design for less energy loss.by resister force=F.*

laminar flow velocity=V.

(c)**Turbulence vanishing rate**:

$\varepsilon$ may be a size in integral volume D=SL=SV $\varepsilon$ ?.

(d)Energy loss/unit time of ship=FV is proportional to velocity$^3$.

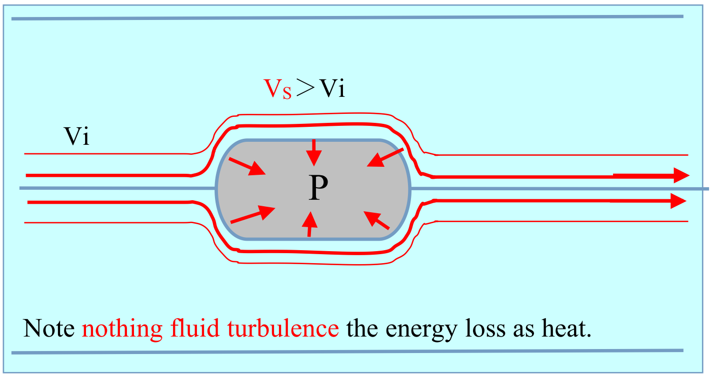
http://www.jpmac.or.jp/img/relation/pdf/pdf-environ-p16-p24.pdf

$F=\mu\nabla^2 V$, thereby also $\mu$ is proportional to V!.

⑵**Bernoulli Theorem(stationary complete flow model):**

A complete fluid is supreme contrary side of fluid turbulence.In a complete fluid flow,there could be 0 resistance force against something symmetric shape( ship,ball,..)in laminar flow (the paradox).Because all surface pressure at uniform stream line velocity flow in constant Ω is just the same as uniform that of static air.

$\frac{1}{2}\rho \mathbf{V}^2 + P + \rho\Omega$ = constant(Energy Conservation Law).




No resister force is due to uniform pressure of stream lines($V_S$) along the ship shape (red line along the ship shape). in a **laminar flow**.This is a paradox against common sense. If Vi is sufficiently small, a laminar flow is realizable.

**Proof on Bernoulli Theorem** :

＊$D\mathbf{V}/Dt \equiv \partial \mathbf{V}/\partial t + (\mathbf{V}\cdot \mathbf{grad})\mathbf{V} = \mathrm{grad}(\mathbf{V}^2/2) - \mathbf{V}\times\mathrm{curl}\mathbf{V}$.　　$f \equiv -\mathrm{grad}\Omega$.

$\rho\, D\mathbf{V}/Dt = \kappa\nabla^2\mathbf{V} - \mathrm{grad}\, P + \rho f$.

Let assume stationary complete fluid of $\kappa\nabla^2\mathbf{V}=0$(curl$\mathbf{V}$=0),and derive energy conservation law.

$\rightarrow \mathbf{V}\cdot \rho(\partial \mathbf{V}/\partial t + \mathrm{grad}(\mathbf{V}^2/2) - \mathbf{V}\times\mathrm{curl}\mathbf{V}) = \mathbf{V}\cdot\{-\mathrm{grad}\, P + \rho(-\mathrm{grad}\Omega)\}$

$\rightarrow \rho\{\mathbf{V}\cdot \partial \mathbf{V}/\partial t + (\mathbf{V}\cdot\mathrm{grad})(\mathbf{V}^2/2)\} = -\{\mathbf{V}\cdot\mathrm{grad})(P + \rho\Omega)\}$

$\rho[(\partial \mathbf{V}^2/\partial t) + (\mathbf{V}\cdot\mathrm{grad})(\mathbf{V}^2/2)] + [(\partial/\partial t) + (\mathbf{V}\cdot\mathrm{grad})](P + \rho\Omega)] = (\partial/\partial t).(P + \rho\Omega)]$

$(D/Dt)[\frac{1}{2}\rho\mathbf{V}^2 + (P + \rho\Omega)] = (\partial/\partial t).(P + \rho\Omega) \equiv 0$. ⟨stationary assumption⟩

$\frac{1}{2}\rho\mathbf{V}^2 + P + \rho\Omega$ = constant in time.

⑶**Random Fluctuation in Non Rotational Soccer Ball Trajectory.**

***Soccer Goal by not normal ball.***

https://www.youtube.com/watch?v=Lbmmq0qHszc

https://www.youtube.com/watch?v=TOUylg7Eyec

☞: Soccer ball curving by ball **rotation(**magnus-effect**)** and ball fluctuating by **no rotation** is different. However those force are due to negative pressure by the surface fluid turbulence. It is a visible evidence of macroscopic random fluctuation realization in fluid dynamics.

It is due to turbulence with $\mu$ .

http://gigazine.net/news/20140619-magnus-effect-world-cup-ball/

[2]:**Dramatic Transition at Viscosity in Fluid Turbulence Generation**.

How much turbulence energy and the air viscosity in soccer ball trace ??.We could coarsely estimate fluid parameters of $\mu$ by observing soccer ball dynamics.Then we could not help to conclude marvelous $\mu$ value transition from laminar flow to turbulence one.

This fact could be **the cause of macroscopic fluctuation in Climate Fluid Dynamics**.

(a)ball dynamic energy loss $\Delta Q$=heat energy by turbulence generation with $\mu$.

$$\Delta[\tfrac{1}{2}m\mathbf{V}^2+mgh]=\Delta Q=\mu\oiiint_D dV\int_0^{\Delta t}dt[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2].......(a)$$

Kinetic Energy Loss of ball$=\Delta Q$=heat energy by turbulence density integral.the volume.

Author has decisive confidence on this energy conservation law relation,so,in the following calculation,he tried to make variable value adjust to fit the energy relation.

(b)Experiment Parameters:

*$\Delta V$=44.4m/s($V_i$=160Km/s)−38..8m/s($V_f$=140Km/s) by d≡20m flight distance.

coarse average flight speed<V>=150Km=41.7m/s.

*$\Delta Q\equiv\Delta(M\mathbf{V}^2/2)\equiv(M\mathbf{V}_i^2/2-M\mathbf{V}_f^2/2)=105$**J.**

*$\Delta mgh=0$.

*ball weight=$M_{ball}$=0.45Kg,

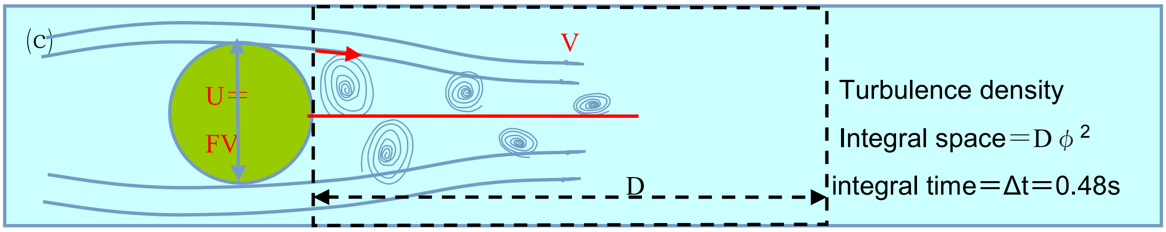
*ball diameter=$\phi$=0.7m/π=0.22m, <V> and this are to determine turbulence space.

*flight time =integral time=Δt=0.48s=20m/150Km/s.

*D=4m???. This is the average length of turbulence tail.

*$D\phi^2$?=This is the integral space volume of turbulence density.

(c)

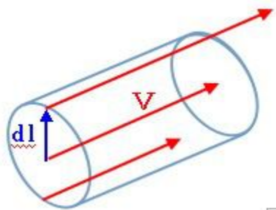



(d)**Very Coarse Estimating** $[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2]\sim$

$*|\mathrm{curl}\mathbf{V}|\equiv d|\mathbf{V}|/dl\equiv|\mathrm{div}\mathbf{V}|$,

*displacement dl is taken to max d$\mathbf{V}$, dl.& d$\mathbf{V}$ are perpendicular.

After all,author without knowledge on turbulence can not help,but take very coarse and assumption. Aim is deriving the order estimation.



(e)$\langle \mathrm{curl}\,V\rangle \equiv \langle \mathrm{div}\,V\rangle \fallingdotseq d|V|/dl \sim \langle V\rangle[\phi/2]$.　　$\phi = 0.22$m

| <V> | <V> | <V/2>[ϕ/2]> | $[(\mathrm{div}\,V)^2+(\mathrm{curl}\,V)^2]=2[\langle V\rangle/\phi]^2\equiv U$ |
|---|---|---|---|
| 160Km | 44.4m/s | 202/s | $8.2\times10^4 s^{-2}$ |
| 150Km | 41.7m/s. | 190/s | $7.2\times10^4 s^{-2}$ |
| 140Km | 38..8m. | 177/s | $6.2\times10^4 s^{-2}$ |

(f)$\langle \oiint_D dV \int_0^{\Delta t} dt\rangle = \phi^2 D\cdot\Delta t = \langle (0.22\mathrm{m})^2\times\{2,5,10\mathrm{m}\}\times(20\mathrm{m}/41.7\mathrm{m/s.})\rangle = \{,0.12,0.23\}\mathrm{m}^3\mathrm{s}$

This was explained in (b).

(g)$\mu = \Delta Q/\langle \oiint_D dV \int_0^{\Delta t} dt\rangle[(\mathrm{div}\,V)^2+(\mathrm{curl}\,V)^2] = \Delta Q/\phi^2 D\cdot\Delta t.U$.

| $\phi^2 D\cdot\Delta t$. | $U=6.2\times10^4 s^{-2}\sim8.2\times10^4 s^{-2}$ | | $\mu$ ($\Delta Q$=105J) ($\mu_t/\mu_L=1.8\times10^{-5}$) | |
|---|---|---|---|---|
| $0.05\mathrm{m}^3\mathrm{s}$ | | | 0.023(1280) | 0.018(1000) |
| $0.12\mathrm{m}^3\mathrm{s}$ | | | 0.014(780) | 0.011(611) |
| 0.23, | | | 0.007(390) | 0.0056(311) |

(h)Air viscosity in rather laminar flow $\mu_L = 1.8\times10^{-5}$ kg/m s.

http://www.engineeringtoolbox.com/dry-air-properties-d_973.html

Above table $\mu$ value is evidently too large from $\mu_L=1.8\times10^{-5}$.

☞ ; This value means (2～3)digit big difference of $\mu$ between laminar and turbulence flow.

(i)$\mu = \Delta Q/\langle \oiint_D dV \int_0^{\Delta t} dt\rangle[(\mathrm{div}\,V)^2+(\mathrm{curl}\,V)^2] = \Delta Q/\phi^2 D\cdot\Delta t.U$.

$\mu = \Delta Q/\phi^2 D\cdot\Delta t.U = \Delta W\Delta t/\phi^2 D\cdot\Delta t.U = \Delta W/\phi^2 D\cdot U$.

☞ ; A flow tends to be stable linear one,not curving one.While fluid must be in full space at anytime.However something is to force curving flow,which needs rotational force with angular momentum generation which is nothing,but turbulent eddy.

A turbulent eddy could not be stable,because rotational flow surface can not accord with the environmental flows,but the destiny is to be terminated as heat at last.

＊Initial and Final Soccer Ball Speed by calculation with Drag coefficient＝$C_D$.

https://en.wikipedia.org/wiki/Drag_coefficient

http://wedge.ismedia.jp/articles/-/3906?page=2

$C_D$＝0.017 is at above site.

$M(dV/dt) = F_D = C_D(A\rho_a V^2/2) = 0.17\mathrm{x}\pi(0.11\mathrm{m})^2\mathrm{x}1.205\mathrm{Kg/m}^3.\mathrm{x}V^2/2$,..... $\rho_a = 1.205\mathrm{Kg/m}^3$.

$dV/dt = V^2\mathrm{x}0.17\mathrm{x}1.205\mathrm{Kg/m}^3.\mathrm{x}\pi(0.11\mathrm{m})^2/2\mathrm{x}0.45\mathrm{Kg} = 0.0087V^2$.

| | | | | | | |
|---|---|---|---|---|---|---|
| $V_l$ | | 100Km/h | 130Km/h | 140Km | 150Km/h | 160Km/h |
| $V_l$ | | 27.8 | 36.1 | 38.9 | 41.7m/s | 44.4 |
| k＝0.017,M=0.45kg | | | | | | |
| <dV/dt>＝$0.017kV_l^2/M$ | | $6.5m/s^2$ | $11m/s^2$ | $13m/s^2$ | $15m/s^2$ | $17.5m/s^2$ |
| ΔV=0.48sx3600sx<dV/dt> | | 10km | 18km | 21km | 24Km | 27km |
| $V_f$ | | 90km | 102km | 119Km/h | 126Km/h | 133km |

＊t=D/V=20m/V=20m/(41.7m/s)=0.48s

⑷Afrter all,μ is **definite** in laminar flow,while random variable in turbulence flow, In laminar flow, $\nabla^2 V=0$, that is,so called **complete fluid** without energy loss eddy. The most difficulty in Fluid Dynamics is at turbulence.Even $\mu$ is something constant,it would cause chaos due to entropy increasing(information loss)by irreversible heat generation in $\mu\nabla^2$**V.** Above all,if $\mu$ is big random over standard value.NS equation could not help to be stochastic equation in climate dynamics!.

⑸**Chaos Model Pattern in Climate Fluid Dynamics Study**.

$\nabla^2$**V**＝graddiv**V**－curlcurl**V.**

Fluid chaos is categorized as two elementary type.However frequent curving(rotational )fluid is both with the forces.So author don't know well significance of the categorization.

①**Jet blow,or sink blow**＝graddiv**V**。 ⇔ frontal,or back ward collision of fluid mass.

ⓐwall collision

ⓑcoast land,mountain collision

ⓒfluid mass collision(at lowest pressure point of cyclone,by cold and hot air mass one)

②**Dragging flow**＝－curlcurl**V.** ⇔ side dragging collision of fluid mass.

ⓐwall(coast) collision

ⓑfluid mass collision(lowest pressure point of cyclone,cold and hot air mass one).

ⓒmass density gradient by that of temperature(local convection flow) in gravity field

ⓓboundary surface dragging(wind collision at sea surface,at mountain surface).

**[3]: Turbulence in Reinolds Number View.**

Our aim is to show cause of **visible macro climate fluctuation** due to Fluid Dynamics(FD).

Most remarkable feature difference between turbulent flow and laminar one may be sudden energy loss increasing in experiments,which is only due to **viscosity term** $=\mu\nabla^2\mathbf{V}$.

(1) $\Delta Q \equiv \int_0^t dt \oiiint_D dV\, \mathbf{V}\cdot\mu\nabla^2\mathbf{V} = \mu\int_0^t dt \oiiint_D dV[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2]$.

The energy loss $\Delta Q$ is also entropy increasing $\Delta S=\Delta Q/T$ <T=temperature of fluid>.

Note the entropy increasing is also information loss,which is nothing ,but the validity of so call CHAOS in FD.It is transition from causalstic to random in MACRO SCALE.

Now we show an evidence of singular behavior of $\mu$ in transition from laminar to turbulent flow by conception of ***Reinolds Number***≡Re.

(2)***Reinolds Number***≡Re:**the physical meanings.**

Re is defined as 0 dimensional by ratio of (physical value/$\mu$).Also Re* is 0 dimensional.

$Re\equiv\rho VL/\mu$.

$* \ \rho[\partial\mathbf{V}/\partial t+\rho(\mathbf{V}\cdot\nabla)\mathbf{V}]=\mu\nabla^2\mathbf{V}-\mathrm{grad}P-\mathrm{grad}\Omega$

$* \ (\mathbf{V}\cdot\nabla)\mathbf{V}=(\nabla\mathbf{V}^2/2)-\mathbf{V}\times(\nabla\times\mathbf{V})$.

$Re^*\equiv\rho(\mathbf{V}\cdot\nabla)\mathbf{V}/\mu\nabla^2\mathbf{V}=\rho(\mathbf{V}\cdot(\mathbf{V}\cdot\nabla)\mathbf{V})/(\mathbf{V}\cdot\mu\nabla^2\mathbf{V})=\Delta\mathbf{K}/\Delta\mathbf{Q}$。

$\Delta\mathbf{K}=\rho(\mathbf{V}\cdot(\mathbf{V}\cdot\nabla)\mathbf{V})=\rho(\mathbf{V}\cdot(\nabla\mathbf{V}^2/2)-\mathbf{V}\times(\nabla\times\mathbf{V}))=\rho((\mathbf{V}\cdot\nabla)\mathbf{V}^2/2))$.

$\Delta\mathbf{K}$ is substantial derivative of kinetic energy density.It may be called flow momentum in FD.By observation,increasing $\Delta\mathbf{K}$ ($\mathbf{V}$→larger)is to cause sudden transition from laminar to turbulence flow,at when energy loss is to show sudden increasing..

$Re^*=\rho((\mathbf{V}\cdot\nabla)\mathbf{V}^2/2))/\mu[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2]$

$\mathbf{V}\equiv k\mathbf{V_0}$.That is V increasing is also k increasing.

$Re^*=(k/\mu)\rho((\mathbf{V_0}\cdot\nabla)\mathbf{V_0}^2/2))/[(\mathrm{div}\mathbf{V_0})^2+(\mathrm{curl}\mathbf{V_0})^2]\equiv(k/\mu)\cdot Re^*_0$.

(a)In laminar flow, Re*=increases by increasing **V.**

(**V**,k→larger) is Re*=increasing in condition $\mu$ =constant.

(b)At larger {$Re^*(k_C)$},when turbulence is to begin with larger heat loss.

We denote at this **after critical point** by {$\mathbf{V_C}, k_C, \mu_C$}.

$\Delta\mathbf{Q}=\mu[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2]\ll\Delta\mathbf{Q_C}=\mu_C[(\mathrm{div}\mathbf{V_C})^2+(\mathrm{curl}\mathbf{V_C})^2]$

$\mathbf{V}+\delta\mathbf{V}=\mathbf{V_C}$、a few velocity **V**= increasing ,while $\Delta\mathbf{Q_C}$ is to generate sudden big heat loss at after critical velocity=$\mathbf{V_C}$ point.

(c) $\mu$ can cause big transition by order jump in the ratio $=10^2, 10^3$ ?。

$\mathbf{V}+\delta\mathbf{V}=\mathbf{V}_C$、 a few velocity $\delta\mathbf{V}$= increasing can not be too much!?.

In soccer ball experiment,we observed big jump of $\mu_C/\mu_0=10^2, 10^3$.

Certainly stirring term$=[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2]$ could become **visible wild**,

however $k_C{}^2/k_0{}^2=10^2, 10^3$ may be impossible ?!.

(d)Reconsideration on $\mu$ original definition only by curl**V.**

In following [4]、we could show important role of jet blow by div**V**.In turbulence generation, not only curl**V**,but also div**V** due to **pressure gradient** becomes decisive.The energy loss by div**V** may be deficit in **original definition** $\mu$ in larminar flow.It is which that needs sudden jump of $\mu$.

(e)Re* is down trend by sudden heat increasing$=\Delta \mathbf{Q}$.

$\mathrm{Re}^*\equiv\rho(\mathbf{V}\cdot\nabla)\mathbf{V}/\mu\nabla^2\mathbf{V}=\Delta\mathbf{K}/\Delta\mathbf{Q}$。

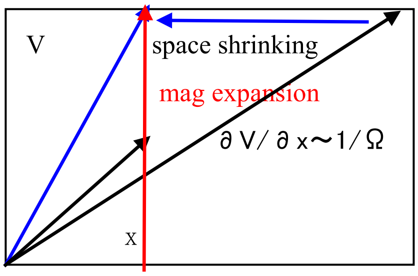
(f)**space factor due to derivative in space**(maginitude{V,k} is fixed)**.**

$\partial/\partial x=j\omega_x$,by Fourier Transform,and we denote $\omega_x/\omega_{x0}\equiv1/\Omega$.

$\partial V(x)/\partial x=(\partial/\partial x)\int d\omega[\exp(j\omega x)U(\omega)]=\int d\omega\, j\omega[\exp(j\omega x)U(\omega)]$

$(\partial/\partial(\Omega x))\int d\omega[\exp(j\omega x)U(\omega)]=\int d\omega\, j(\omega/\Omega)[\exp(j\omega x)U(\omega)]$

That is,stronger spatial derivative is smaller Ω.It is space shrinking with keeping **V** magnitude. Or it is keeping Ω,while expanding **V** magnitude.That is, equivalent to velocity(k) increasing




So we derive,

$\mathbf{V}\cdot\rho(\mathbf{V}\cdot\nabla)\mathbf{V}\sim1/\Omega$ kinetic energy term.

$\mu\mathbf{V}\cdot\nabla^2\mathbf{V}\sim\mu/\Omega^2$. heat loss term

$*\mathrm{Re}^*=(k/\mu)\mathrm{Re}^*_0$. the original definition.

$\mathrm{Re}^*\equiv\rho(\mathbf{V}\cdot\nabla)\mathbf{V}/\mu\nabla^2\mathbf{V}=(\langle k/\Omega\rangle/\langle\mu/\Omega^2\rangle)\mathrm{Re}^*_0.=(k\Omega/\mu)\mathrm{Re}^*_0$.

That is,stronger spatial derivative for velocity is larger Ω,

which is to increase Re* toward turbulence.Stirring term$=[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2]$ could become visible wild by intensifying spatial derivative.

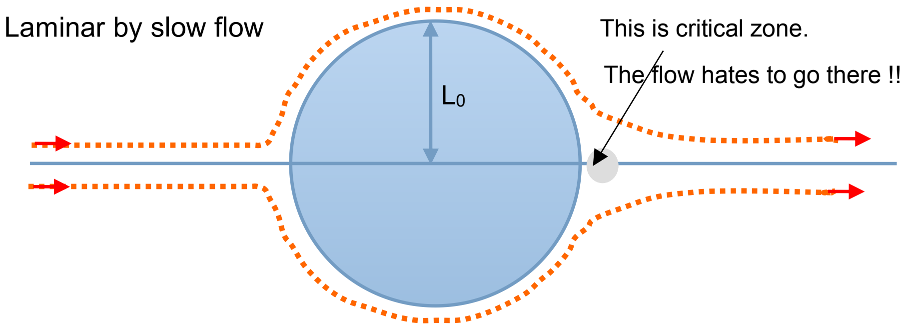
(3)Turbulence may be caused by intensifying both μ and stirring term.

[4] :**Genesis of Turbulence**<breaking down of laminar flow by emerging something stop>.

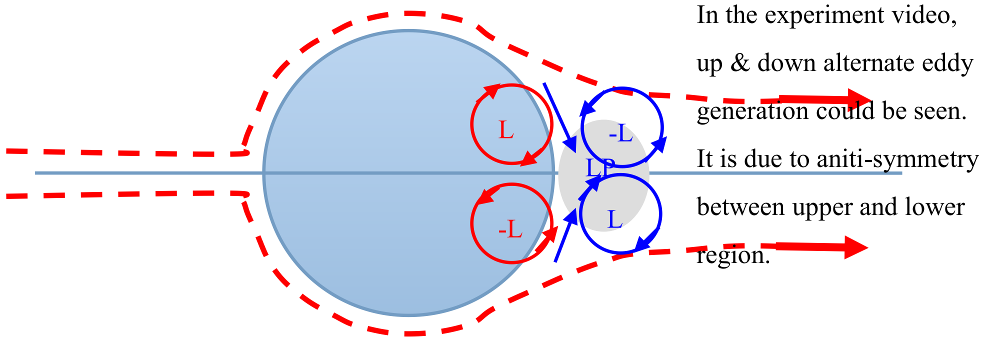
https://www.youtube.com/watch?v=TOUylg7Eyec

(1)A laminar flow is ensemble of **local straight line and smooth flow without eddy one** with almost same velocity.Then rapid laminar flow becomes impossible due to emerging something stop(＝crashing,strong friction force by contacting something).

**Logical negation** of straight flow is curving(rotational)one.Note turbulence is ensemble of random eddy-s.To generate many eddy,there must be generation of **angular momentum** which could be generated by rotational force curving linear flow by something stop.




Critical laminar rapid flow with visible turbulence initiation.




In order to inject flow at the critical zone,laminar flow must be curved by rotational force＝**N**＝d**L**/dt,where **L** is angular momentum.Then **conservation law of angular momentum** needs －**L**,which is to generate eddy flows(upper half plane,while lower one is negative). This is origin of visible(**macroscopic**)large eddy turbulence＝$(\mathrm{curl}\,\mathbf{V})^2$.

Also note critical zone is hated as laminar flow rejecting flowing into there(2 blue arrows). Thereby the zone is lower pressure(＝LP),of which pressure=P gradient is to cause jet blow ＝$(\mathrm{div}\,\mathbf{V})^2$.

＊ div**V**＝－D(lnP)/Dt＋D(lnT)/Dt～－(**V**· gradP). substantial derivative with P.

The details are to mention in below.

(2)Thus,heat energy$=\Delta Q$ becomes larger in turbulence,which is also entropy increasing.

This is also information deficit=randomness generation=**origin of fluid chaos !!**

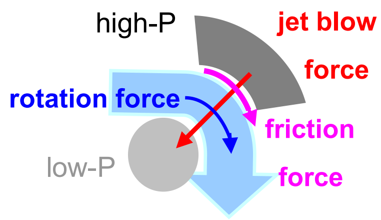
$$\Delta Q \equiv \int_0^t dt \oiint_D dV\, \mathbf{V}\cdot\mu\nabla^2\mathbf{V} = -\mu\int_0^t dt \oiint_D dV[(\mathrm{div}\mathbf{V})^2+(\mathrm{curl}\mathbf{V})^2].$$

(3) $(\mathrm{curl}\mathbf{V})^2$ : :

Once some portion had become turbulent,which shall interfere with near stream lines, which turn to intensify more growing turbulence<self reproducing>.It is positive feedback mechanism similar with multi car crashing in high way.Thereby it becomes rapid exponential increasing toward visible wild turbulent flow<laminar to turbulent flow sudden transition>.

(4) $(\mathrm{div}\mathbf{V})^2$ : :

Rapid Turbulent stream lines will not curve.
If having forced to curve,outer surface of curve of semi circle becomes higher pressure, while that of inner surface becomes lower one which causes pressure gradient.

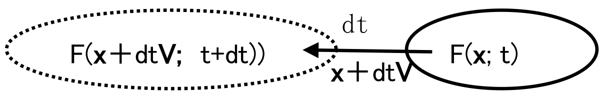



<also see **generalized pressure gradient** at here APPENDIX_4>

＊substantial derivative in fluid dynamics field.

$DF/Dt \equiv \partial F/\partial t + (\mathbf{V}\cdot \mathrm{grad}F)$......total derivative=partial one+substantial one.

$DF/Dt \equiv \{F(\mathbf{x}+dt\mathbf{V};\ t+dt)-F(\mathbf{x};\ t)\}/dt.$




*Ideal gas state equation.*

$*P=(\rho/M)RT. \rightarrow\ \rho=PM/RT. \rightarrow\ \ln\rho=\ln P-\ln T+\ln(M/R) \rightarrow\ \ln\rho=\ln P-\ln T+\ln(M/R).$

*mass density conservation law.*

$\partial\rho/\partial t=-\mathrm{div}(\rho\mathbf{V})=-\mathrm{V.grad}\rho-\rho\,\mathrm{div}\mathbf{V}. \rightarrow\ \mathrm{div}\mathbf{V}=-\partial_t\ln\rho-\mathrm{V.grad.}\ln\rho=-D\ln\rho/Dt.$

$\rightarrow\ \mathrm{div}\mathbf{V}=-D(\ln P)/Dt+D(\ln T)/Dtt\sim-(\mathbf{V}\cdot\ \mathrm{grad}P).$ substantial derivative with P.

That is,**jet blow** is caused by pressure and temperature(explosion)time change(**substantial derivative in stationary flow**) which is caused by bending rapid stream lines.
Thus $(\mathrm{div}\mathbf{V})^2$ and $(\mathrm{curl}\mathbf{V})^2$ are not separated ones,but may be co-body in turbulence.

After all,turbulent eddy is to be absorbed in massive momentum of laminar flow. Turbulent eddy flow destiny is to vanish ,because the surface around eddy volume could not be flows according with the eddy rotation.If rotation is counter wise,eddy angular momentum energy is to be absorbed to be heat at last.It is nothing,but a vanishing of fluid macro trace !!.

**APPENDIX_1：Not initial data Error,but the Evolution Equation Jitter.**

What author wish to tell is as follows.**Actual FD** is **stochastic equation** with random variable **μ**=viscosity due to turbulence which may depend on (t;x).In other world,climate prediction FD is **deterministic**,but not stochastic.That is,error difference between reality and calculation theory is to occur due to **the evolution equation itself's error(jitter)**. Even we took observed actual data(very many variables due to many observed field points at initial time=$t_0$ ),those could not fit the actual FD at time=$t_0$,but be with errors.On the contrary,according to **Ensemble Prediction Method**,one or two ? of data in **randomized Initial data ensemble** could hit actual prediction.

Following are quasi-fluid dynamics description by view on fluid particle trace.

(1)**From Euler View to Lagrange One**.

Fluid Equation(FD) in *Euler View* do not pursue fluid particle trajectory,but ovserve fixed,but all space point velocity at a time.However **ordinal style of Newton Equation**(NE) must be exist uniquely as *Lagrange View*,while author don't know the explicit form.

①$D(\rho(t;x)\mathbf{V}(t;x))/Dt \equiv \partial(\rho(t;x)\mathbf{V}(t;x))/\partial t+(\mathbf{V}\cdot\ \mathrm{grad})(\rho(t;x)\mathbf{V}(t;x))=\mathbf{f}(t;x)$. **<** *Euler View>*

②$d(\rho(t;x_0)\mathbf{V}(t;x_0))/dt \equiv d(\rho(t;x_0)d\mathbf{x}(t;x_0)/dt)/dt=\mathbf{F}(t;\mathbf{x}(t;x_0))$. **<** *Euler View>*

$\mathbf{x}(t;x_0) \equiv$ **fluid particle position** at time=t with initial position=$x_0(t_0)$.

$\mathbf{V}(t;x_0)) \equiv d\mathbf{x}(t;x_0)/dt$.

(1)NE's the most difficulty is that force=**F** must be function with unknown varialble $\mathbf{x}(t;x_0)$, It is far more non linearity than that of substantial derivative=$(\mathbf{V}\cdot\ \mathrm{grad})(\rho(t;x)\mathbf{V}(t;x))$. This may be the reason why anyone will not employ NE in fluid dynamics.

(2)**NE by force with random variable=** $\delta\mu$ **((Climate Prediction View Point))**.

$d(\rho(t;x_0)\mathbf{V}(t;x_0))/dt=\mathbf{F}(t;\mathbf{x}(t;x_0);\mu_0+\delta\mu(\mathbf{x}(t;x_0)))=\mathbf{F}(t;\mathbf{x}(t;x_0);\mu_0)+\delta\mu\,(\partial\mathbf{F}/\partial\mu)$.

$\delta\mathbf{V}(t;x_0))=\rho(t;x_0)^{-1}\int_{t0}^{t}dt.[\delta\mu\,(\partial\mathbf{F}/\partial\mu)]$=fluctuation due to random force.

☞:So long as we observe laminar flow in good days sky,it would be laminar flow for the time being(no strong turbulence),while we observe bad weather,sky is many of rapid variations of cloud with something fall and windy(observing turbulence).Also author had become aware that recent years weather prediction sometimes make wrong.Especially long times prediction seems difficult to make exact prediction.

Then how much initial data error would enlarge in future prediction?.

(3)**variation due to initial condition one.**

$x(t)=x(x_0(t_0);t)=\int_0^t dt V(x_0;t)+x_0$.

$x=x(x_0+\delta x_0;t)=\int_0^t dt V(x_0+\delta x_0;t)+x_0$

$=x_0+\int_0^t dt V(x_0;t)+\int_0^t dt\{\delta x_0\cdot\partial V(x_0;t)/\partial x_0\}$

$=x(x_0(t_0);t)+\int_0^t dt\{\delta x_0\cdot\partial V(x_0;t)/\partial x_0\}$

$=x(x_0(t_0);t)+\delta x_0\cdot\langle\int_0^t dt\{\partial V(x_0;t)/\partial x_0\}\rangle\equiv x(x_0(t_0);t)+\delta x(\delta x_0+x_0(t_0);t)$

$*\ \delta x_0\cdot\partial V(x_0;t)/\partial x_0\equiv\langle\delta x_0\cdot\mathrm{grad}\rangle V(x_0;t)$.substantial derivative as for $x_0$.

$\delta x_0\cdot\partial V(x_0;t)/\partial x_0=\delta x_0\cdot(d/dt)\partial x(x_0;t)/\partial x_0=(d/dt)[\delta x_0\cdot\partial x(x_0;t)/\partial x_0]\equiv(d/dt)\delta x(x_0;t)$.

$\delta x_0\cdot\partial x(x_0;t_0)/\partial x_0\equiv\delta x(x_0;t_0)=\delta x_0.\to\partial x(x_0;t_0)/\partial x_0=1$.<initial variation definition>.

(4)Taylor expansion of the initial variation:

$\partial x(x_0;t)/\partial x_0=1+t[\langle\partial/\partial t\rangle\partial x(x_0;t_0)/\partial x_0]+(t^2/2)[\langle\partial/\partial t\rangle^2\partial x(x_0;t_0)/\partial x_0]++$

$(d/dt)\delta x(x_0;t)=\delta x_0[\langle\partial/\partial t\rangle\partial x(x_0;t_0)/\partial x_0]+\delta x_0 t[\langle\partial/\partial t\rangle^2\partial x(x_0;t_0)/\partial x_0]+$

$\delta x(\delta x_0+x_0(t_0);t)\equiv\delta x_0\cdot\langle\int_0^t dt\{\partial V(x_0;t)/\partial x_0\}\rangle$

$=\delta x_0\{t[\langle\partial/\partial t\rangle\partial x(x_0;t_0)/\partial x_0]+(t^2/2)[\langle\partial/\partial t\rangle^2\partial x(x_0;t_0)/\partial x_0]+\dots\dots\}$

$=\delta x_0\{t[\partial V(x_0;t_0)/\partial x_0]+(t^2/2)[\langle\partial/\partial t\rangle\partial V(x_0;t_0)/\partial x_0]+$

(5)For example,**uncertainty of cyclone trace** is almost proportional time<prediction cone>.

That is,the main cause is due to $[\partial V(x_0;t_0)/\partial x_0]$.

**Stronger space variation of velocity** at initial time-position is to cause larger position variation in future.This may agree with experienced law in weather prediction.

## APPENDIX_2 : Quasi-Diffusion Equation.

### (1)Newton Equation as Causalstic Dynamic One.

Newton Equation is derived from **statistical averaging** in Schródinger one.

<Ehrenfest Theorem in Quantum Mechanics>.

**Quantum Mechanics** is the most exact material science,then it conclude **Newton Mechanics** as the averaging meaning,It is complete causalstic. Sometimes,such Newton Mechanics can give good insight even in microscopic molecular dynamics.

＊$\mathbf{p}=-i\hbar\,\mathrm{grad}$

＊$i\hbar\partial\Psi(x;t)/\partial t=\mathbf{H}(x)\Psi(x;t)=[\mathbf{p}^2/2m+U(\mathbf{x})]\Psi(x;t)=[-\hbar^2\nabla^2/2m+U(\mathbf{x})]\Psi(x;t).$

$(d/dt)\langle x(t)\rangle\equiv(d/dt)|\int dx^3\Psi^*(x;t)x\Psi(x;t)|=(1/m)\int dx^3\langle\Psi^*(-i\hbar\,\mathrm{grad})\Psi\rangle=\langle\mathbf{p}/m\rangle.$

$(d/dt)\langle\mathbf{p}\rangle=\int dx^3\langle\Psi^*(-\mathrm{grad}U(\mathbf{x}))\Psi\rangle=\langle-\mathrm{grad}U(\mathbf{x})\rangle=\langle F\rangle$...... **Newton Equation.**

＊citation,L.Schiff,Quantum Mechanics.Magraw hill,1965.

Note:it is important enough to derive causalstic Newton Dynamic Equation by averaging.

**Averaging operation could derive something causalstic mechanism.**

This fact become very useful in climate prediction in random data.

### (2)Langevin Stochastic Equation(Brownian Motion).

$(d/dt)^2\mathbf{x}(t)=-(\mu/m)(d/dt)\mathbf{x}(t)+\mathbf{R}(t)/m\equiv-\eta\,d\mathbf{x}(t)/dt)+\mathbf{R}(t)/m$..

2nd term is viscosity force and 3rd is random force.That is,the <**Tme and Ensemle Average**> is zero,where $0=\langle\mathbf{R}(t)/m\rangle$.　Now we are to derive $(d/dt)\langle\mathbf{x}^2(t)\rangle$

→ $\mathbf{x}(d^2\mathbf{x}/dt^2)=-\eta\,\mathbf{x}(d\mathbf{x}/dt)+\mathbf{x}\mathbf{R}(t)/m.$

＊$(d/dt)^2[\mathbf{x}^2]=(d/dt)[2\mathbf{x}(d\mathbf{x}/dt)]=2\mathbf{x}(d^2\mathbf{x}/dt^2)+2(d\mathbf{x}/dt)^2$

$(1/2)(d/dt)[(d/dt)\mathbf{x}^2]=(d\mathbf{x}/dt)^2-(1/2)\eta(d/dt)[\mathbf{x}^2]+\mathbf{x}\mathbf{R}(t)/m.$

$(1/2)(d/dt)[(d/dt)\mathbf{x}^2]+(1/2)\eta[(d/dt)\mathbf{x}^2]=(d\mathbf{x}/dt)^2+\mathbf{x}\mathbf{R}(t)/m.$

At here,we take *Ensemble Averaging* to vanish $\mathbf{R}(t)$. Also $\Xi\equiv\langle(d/dt)\mathbf{x}^2\rangle$.

$\langle m(d\mathbf{x}/dt)^2\rangle/2\equiv kT/2$ is a result of statistcal mechanics,

→ $(d\Xi/dt)+\eta\,\Xi=2kT/m.$

→ $\Xi(t)=2kTt/m\eta+C\exp(-\eta t)=2kT/m\eta.\ (t\to\infty).$

→ $\langle\mathbf{x}^2\rangle=2(kT/m\eta)t\equiv2Dt.$　Random Walk.

☞:Deviation is proportional to time,this is peculiar feature of random walk.

＊citation,Kyoritsu publishing Co,Collection on Physics Formulae,Tokyo,1970

(3)**Evolution Equation from Normal Distribution with time dependent average=<x(t)>.**

(a)Following is a trial to derive evolution equation from Normal Distribution with time dependent average=<x(t)>.Our concern is view on mixed effect of random term and causalstic one.

$\partial\rho/\partial t=\kappa\nabla^2\rho.\quad\rightarrow\quad \rho(\mathbf{x};t)=\rho_0(4\pi\kappa t)^{-1/2}.\exp[-(x-<x(t)>)^2/4\kappa t].$

$\partial\rho/\partial t=-(1/2)\rho_0(4\pi\kappa)^{1/2}t^{-3/2}\exp[-(x-<x(t)>)^2/4\kappa t]$

$+\rho_0(4\pi\kappa t)^{-1/2}.\exp[-(x-<x(t)>)^2/4\kappa t]\times\{[(x-<x(t)>)^2/4\kappa t^2]$

$+\rho_0(4\pi\kappa t)^{-1/2}.\exp[-(x-<x(t)>)^2/4\kappa t].\ [-2(x-<x(t))/4\kappa t]\}\partial/\partial t)<x(t)>$

$*((\partial/\partial x)\rho_0(4\pi\kappa t)^{-1/2}.\exp[-(x-<x(t)>)^2/4\kappa t]$

$=\rho_0(4\pi\kappa t)^{-1/2}.\exp[-(x-<x(t)>)^2/4\kappa t].[-2(x-<x(t)>)/4\kappa t]$

$**(\partial/\partial x)^2\rho_0(4\pi\kappa t)^{-1/2}.\exp[-(x-<x(t)>)^2/4\kappa t]$

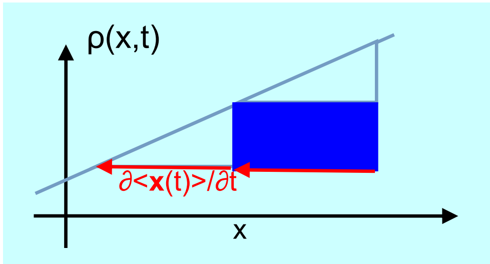
$=\rho_0(4\pi\kappa t)^{-1/2}.\exp[-(x-<x(t)>)^2/4\kappa t].[-2(x-<x(t)>)/4\kappa t]^2$

$+\rho_0(4\pi\kappa t)^{-1/2}.\exp[-(x-<x(t)>)^2/4\kappa t].[-1/2\kappa t]$

$\exp[-(x-<x(t)>)^2/4\kappa t].[-1/2\kappa t]$

$\rightarrow\quad \partial\rho/\partial t=\kappa\nabla^2\rho+\mathrm{grad}\rho.\partial<\mathbf{x}(t)>/\partial t).$

$\rightarrow\quad \partial<\mathbf{x}(t)>/\partial t=<\mathbf{V}(t)>=<\int dt.\mathbf{f}(t)/m>$

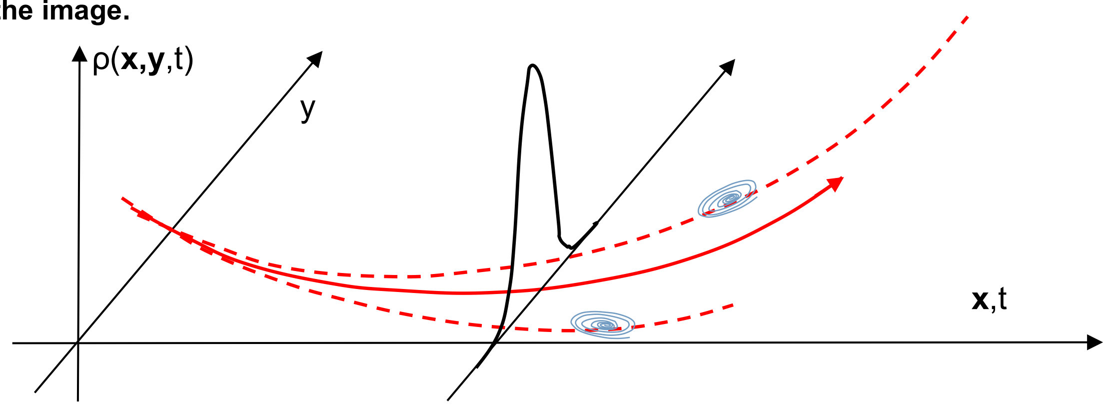



The added term may means **blue box height flow** due to something coherent force causing $\partial<\mathbf{x}(t)>/\partial t$. It might be physically realizable mechanism<**diffusion flow in fluid dynamics**>.

$\rightarrow\quad D\mathbf{V}/Dt=\kappa\nabla^2\mathbf{V}+\mathbf{F}.$

(b)**the image.**

≈




(c)**Global view, but not only microscopic one as random walk.**

A local **laminar flow** mass may be stable for the time being,however as time goes on,the **mass surface** is to encounter different flow with 0≠curl**V**、0≠div**V**,which is to cause turbulence flow(eddy generation). **Whole flow field is to fluctuate(from initial prediction)by micro random accumulation in long time process and by interaction in whole global space.**

http://www.777true.net/Information-Loss-Process-in-NS-Equation_The-Cause-of-Chaos.pdf

## APPENDIX_3 : Spiral Flows Growing Mechanism<Cyclone and Hurricane> .

http://fluid.nuae.nagoya-u.ac.jp/ lecture/nensei08.pdf#search='%E4%B9%B1%E6%B5%81%E3%81%AF%E3%81%AA%E3%81%9C%E8%B5%B7%E3%81%8D%E3%82%8B'

(1)**formulae in vector analysis**<see text book on vector analysis>.

$* \nabla \times V \equiv \omega$ ; $* (V\cdot\nabla)V = (\nabla V^2/2) - V\times\omega$ ; $* \nabla\times(V\times\omega) = (\omega\cdot\nabla)V - (V\cdot\nabla)\omega$.

(2)**fluid equation to eddy equation conversion.**

$\rho[\partial V/\partial t + (V\cdot\nabla)V] = \mu\nabla^2 V - \nabla p$. . . . . . fluid equation in local.

$\nabla\times\{\rho[\partial V/\partial t + (\nabla V^2/2) - V\times\omega]\} = \nabla\times\{\mu\nabla^2 V - \nabla p\}$. $\rho[\partial\omega/\partial t - \nabla\times V\times\omega] = \mu\nabla^2\omega$.

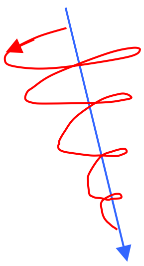
(3)**eddy equation**:

$\rho[\partial\omega/\partial t + (V\cdot\nabla)\omega] \equiv D\omega/Dt = (\omega\cdot\nabla)V + \mu\nabla^2\omega$. 〈stretching term in eddy〉

$D\omega/Dt \equiv$ eddy $\omega$ growing rate

= force by parallel degree intensity between $\omega$ and grad $V$ $+ \mu\nabla^2\omega$ (frictional term).

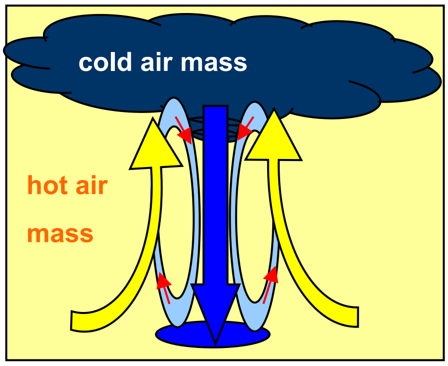
(4)**physical interpretation of *streching term* and cold downburst initiation**:

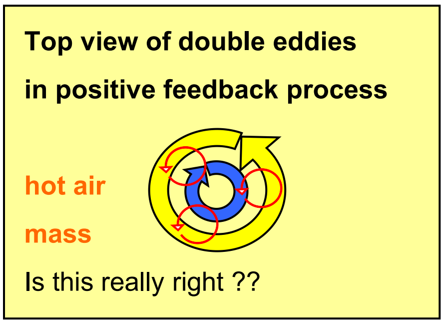


$(\omega\cdot\nabla)V \sim (\omega_z\cdot\partial V_z/\partial z)$.

In left fig, $\omega_z$ orients -z axis,while velocity $V_z$ is grow to -z axis,then force $(\omega_z\cdot\partial V_z/\partial z) > 0$. Growing is a **positive feedback process** due to property of eddy equation-itself. $\mu\nabla^2\omega$ may be negative action.

This scheme could be seen in tornado Initiation from **cold cloud** colliding **warmer air mass**,from where cold down-burst spiral flow begin !.




(5)**growing of strong outer upwelling**

fluid flow must be **closed circuit** due to current volume conservation low.Thereby,cold downburst flow must be compensated by warm upwelling flow.Those may forms double eddy both in inner circle and outer circle,which are **consistent with** (c) **eddy equation**.

(6)**collision of cold and warm air mass.**

This is a phenomenon due to **heat engine** between **cold and warmer air mass collision**,so frequency of tornado generation would be increased by **global warming** which accelerate both equator and Arctic warming.Then also those accelerates **heat exchanging between those**,which is nothing ,but collision of cold and warm air mass.

http://www.afpbb.com/article/environment-science-it/science-technology/2945715/10785642

**APPENDIX_4：Generalized Bernoulli Theorem.** 2010/10/1

(1)**Vector Analysis Formulate.**

$*(\mathbf{V}\cdot\nabla)\mathbf{V}=(\nabla\mathbf{V}^2/2)-\mathbf{V}\times(\nabla\times\mathbf{V})$.

$*\ \Delta\mathbf{V}=\mathrm{graddiv}\mathbf{V}-\mathrm{curlcurl}\mathbf{V}$.

$*(\mathbf{V}\cdot\nabla)\mathbf{V}=(\nabla\mathbf{V}^2/2)-\mathbf{V}\times(\nabla\times\mathbf{V})$

(2)**Fluid Dynamic Equation(≡FD) in Non compressible and non stationary flow**.

$\rho\,\partial\mathbf{V}/\partial t+\rho(\mathbf{V}\cdot\nabla)\mathbf{V}=\rho\,\partial\mathbf{V}/\partial t+\rho(\nabla\mathbf{V}^2/2)-\rho\mathbf{V}\times(\nabla\times\mathbf{V})=\mu\,\Delta\mathbf{V}-\mathrm{grad}P-\mathrm{grad}\,\Omega$.

$\rho\,\partial\mathbf{V}/\partial t-\rho\mathbf{V}\times(\nabla\times\mathbf{V})=\mu\,\Delta\mathbf{V}-\mathrm{grad}P-\mathrm{grad}\,\Omega-\mathrm{grad}(\mathbf{V}^2/2)$.

$\rho\,\partial\mathbf{V}/\partial t-\rho\mathbf{V}\times(\nabla\times\mathbf{V})=-\mu\,\mathrm{curlcur}\mathbf{V}+\mu\,\mathrm{graddiv}\mathbf{V}-\mathrm{grad}P-\mathrm{grad}\,\Omega-\rho\,\mathrm{grad}(\mathbf{V}^2/2)$.

(3)**Rotational Force and Laminar (?)Force Separation.**

$\rightarrow\ \rho(\partial\mathbf{V}/\partial t-\mathbf{V}\times\boldsymbol{\omega})=-\mu\,\mathrm{curl}\,\boldsymbol{\omega}-\mathrm{grad}\,\Xi$.

$=$ **frictional＋pressure**

$*\Xi=\rho\mathbf{V}^2/2+P+\Omega-\mu\,\mathrm{div}\mathbf{V}$. **fluids frontal contacting pressure force**

$*\boldsymbol{\omega}\equiv\mathrm{curl}\mathbf{V}$. **fluids parallel contacting frictional force**

$*-\mathrm{grad}\,\Xi\equiv\mu\,\mathrm{graddiv}\mathbf{V}-\mathrm{grad}P-\mathrm{grad}\,\Omega-\rho\,\mathrm{grad}\mathbf{V}^2/2)$.

$\rho\mathbf{V}.\partial\mathbf{V}/\partial t=-\mu\mathbf{V}\mathrm{curl}\,\boldsymbol{\omega}-\mathbf{V}\mathrm{grad}\,\Xi$.

(4)**Generalized Bernoulli Theorem.**

$\rightarrow\Xi=\rho\mathbf{V}^2/2+P+\Omega-\mu\,\mathrm{div}\mathbf{V}$.

$*\Delta\,\Xi=-\rho\,\mathrm{div}(\partial\mathbf{V}/\partial t-\mathbf{V}\times\boldsymbol{\omega}).\rightarrow$ **pressure force**

$\rho\,\mathrm{curl}(\partial\mathbf{V}/\partial t-(\mathbf{V}\times\boldsymbol{\omega}))=-\mu\,\mathrm{curlcurl}\,\boldsymbol{\omega}$. **frictional force**

$\rightarrow\rho\,\partial\mathbf{V}/\partial t-\rho\mathbf{V}\times\boldsymbol{\omega}=-\mu\,\mathrm{curl}\,\boldsymbol{\omega}-\mathrm{grad}\,\Xi$.

$\rightarrow\ \Delta\,\Xi=-\rho\,\mathrm{div}(\partial\mathbf{V}/\partial t-\mathbf{V}\times\boldsymbol{\omega}).\ \rightarrow$

$\rightarrow\ -\mu\,\mathrm{curlcurl}\,\boldsymbol{\omega}=\rho\,\partial\boldsymbol{\omega}/\partial t-\mathrm{curl}(\rho\mathbf{V}\times\boldsymbol{\omega})$.

**APPENDIX_5 : Macroscopic View on Fluid Dynamics and the Fluctuation Cause.**

Following is Fluid Equation as global surface=S.

∯dV·D(ρv)/Dt＝－μ∯dS×curlV＋[μ∯dS·divV＋∯dS·P]＋∰dV·ρ**K**≡①+②+③.

③**K**＝g＋2V×ω. This is evident as entirely causalitical body forces(gravity+,Colioris).

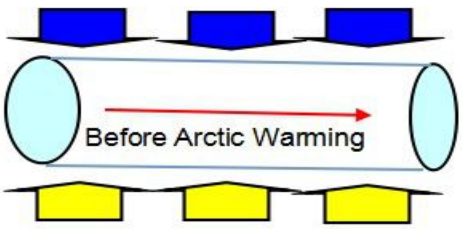
①② are complicated surface forces with rather **random** variable V(x;t).

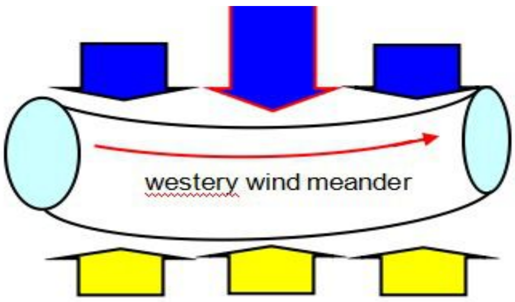
However it could be summarized as pressure force＝∯dS·P in very global view.

Following model is westerly wind meander caused by collision both by cold and hot air mass.

After all,it is due to stronger inhomogeneous-ness in those down and up **laminar flows**,

By global warming,both Arctic and Equator had become warmed.Then **equator convection flow intensity** is to grow with also growing the fluctuation due to heat distribution randomness increasing.,while convection flow intensity becomes down in warmer Arctic.Because it is sink by coldness.As the consequence **Polar Cell Circulation Radius** become larger,which is nothing, but **downward fluctuated cold flow emerging**.



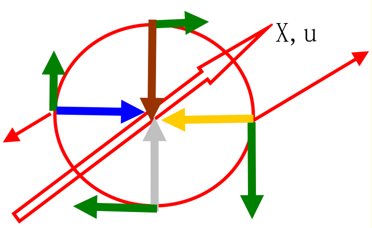

*following is force calculation of spiral flow by substantial derivative..

V＝[u, vcos(ωt-kx), -wsin(ωt-kx)], → DV/Dt＝[0,－ωvsin(ωt-kx),－ωwcos(ωt-kx)]

＋[u∂$_x$＋vcos(ωt-kx)∂$_v$-wsin(ωt-kx)∂$_w$]·[u, ωvcos(ωt-kx),－ωwsin(ωt-kx)]

＝[0,－ωvsin(ωt-kx),－ωwcos(ωt-kx)]＋[0,ukωvsin(ωt-kx),ukωwcos(ωt-kx)]

＝[0, (uk-1)ωvsin(ωt-kx), (uk-1)ωwcos(ωt-kx)]＝F (force causing rotation).




It is told air mass **valley** caused by cold and warm air mass wall contact that enables **spiral flow**(eastward traveling with rotation) The rotation needs **central forces**,Black is gravity downward, Yellow is **top wall pressure** by warmer air mass.Blue is that of cold. Gray is bottom wall force where cold and hot contacting. Red is Colioris force for up(slow) and down(rapid)velocity,

## APPENDIX-6:Q&A on Climate Uncertainty Dynamics(and the Prediction).

**❶What is non-random(causalstic)? in time axis problem.** <2016/10/17>

Those are categolized **stable**(periodic around certain stationary value) and **unstable** **(**divergence to increase or decrease toward catastrophic point or freezing one).The former typical example is mammalian body temperature,the latter may be **global temperature** without fixing toward Arctic Methane Catastrophe. Behind those phenomena,something **casalstic dynamics** has been acting on.Theory must explain the dynamics.

**❷What is random(non causalstic)? in time axis problem.**

No accurate prediction due to **nothing causalstic mechanism**,but certain probability for event $X_k = p_k(X_k)$,,{,k=1,2,...,N}. However the averaging value of ensemble $\delta X_k \equiv (X_k - \langle X \rangle)$must exactly be zero,so those are to fluctuate around the average value=<X>.

**❸How to measure randomness ?<Shannon and Brillouin>.**

$S = k_B \Sigma_{k=1}{}^{,N} \ln(1/p_k)$ is called **entropy**(thermo-statistical dynamics and information theory),which could be measure of randomness.For example,$p_k = 1$,others are all zero, Then $S = 0$.If $p_k = 1/N$ for all k,then $S = N.\ln(N)$ is **maximum value**.The former is certain phenomena,while the latter is **most uncertain phenomena due to equal probability**.

Another simple measure is deviation : $\sigma^2 \equiv \langle X_k{}^2 - \langle X \rangle^2 \rangle$.A rather certain phenomena is small $\sigma^2$,while uncertain one is larger.Especially frequent emerging probability density function(PDF) is **Gaussian**,which is completely determined average value <X> with $\sigma^2$.

**❹How to determine** $\sigma^2$**?**=author don't know well !.**However this is very important !!.**

The most primitive model toward Gaussian is so called **Binominal Distribution**.

1step probability is p,1 back-step is q,then,in N trials,the r step、$P(r) = {}_N C_r\, p^r q^{(N-r)}$.

$\langle r \rangle = Np$. $\sigma^2 = Npq$.,where pq is **go & back probability**,the max value is N/4(p=q=1/2).

That is, **"deviation is a measure for total trial times!!".**Following are examples.

$\sigma^2 \equiv \langle (x - \langle x \rangle)^2 \rangle = 2\kappa t$. **Deviation Time Dependency in Diffusion(Random walk).**

Certainly trial times is proportional to time=t in irreversible process.

$\sigma^2 \equiv \langle V^2 \rangle = 2k_B T/m$. **Deviation Temperature Dependency in Maxwell Distribution**

$T = 2[m\langle V^2 \rangle/2]/k_B$.Temperature is proportional to heat energy of a particle.If we inject heat as time=t,that is T=Wt, $\sigma^2 \equiv \langle V^2 \rangle = 2k_B Wt/m$. This is just same as random walk.

Climate Change is nothing,but just heat energy injecting to planet earth.Thereby **deviation ＝fluctuation in climate variables** such as temperature,wind intensity rain fall,etc are to be exited stronger !!?.

**❺What is critical point and phase transition ??**

Boiling water becomes visible vapor from liquid at 100℃ in 1atm.Ice becomes water from solid at 0℃ in 1atm.Cyclone is generated at sea surface temperature [27～31℃].If Arctic ice extent loss in summer is enough small,those could be recovered enough again in winter, however the loss becomes over a critical point extent(author don't know well),opened sea mouth could absorb more heat which turn to accelerate more ice extent loss.Then it becomes positive feedback,as for which non natural phenomena could stop,but man made Arctic Cooling Engineering.

Those change from liquid to gas,or solid to liquid, change from stable laminar atmospheric flow to organized gigantic wild eddy flow(also by positive feedback mechanism),ice extent which enable positive feedback are called phase transition.Then the specified temperature is called **critical point** temperature(Ice extent). It is visible macroscopic scale change of invisible massive molecular particles.Or it is dramatic change of ice extent toward massive methane(the strong GHG) melting by sea flor temperature rise.

Then statistical mechanists tell that, at the critical point,growing larger fluctuation is to do important role in changing from micro scale to macroscopic one (it is a revolution of regime change,which needs total remaking,but not local remaking).

*Note narrow width thinker may be conservative,while wider width thinker might become revolutionist or bad conspirator.It is told fluctuation in thinking has important role.

In critical point view,climate change is not gradual process,but could be **sudden big disasterous change at critical points,** if nothing counter measure..

The plausible debating on *below 2℃ in COPs(IPCC)* is decisively defect at this view points.

## ❻How to make exact Climate Prediction /!!!.

⑴Exact study on **average trend of** <T(t)>. This must be dead certain.

⑵Exact study on **deviation trend** of $<\Delta T^2(t)>$.

PDF(ΔT(t)) is Gaussian in the meaning of central limit theorem. However the observed result is effected by outer planet elements(**insolation jitter cycles**).